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Contents

Preface

This volume edited by T.F. Kamalov is based on the selected contributions at the 2-nd International Conference on Theoretical Physics which was held at the Moscow State Open University (MSOU) on July 2-6, 2012. The talks on Quantum Theory and quantum informatics presented at this conference are published in the special volume of Journal of Quantum Computers and Computations.

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8-Spinor Field Model and Quantum Mechanics

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Abstract. In the paper the structure of the chiral 8-spinor field model based on the Brioschi identity and its connection with Quantum Mechanics are discussed. The existence of soliton solutions in this model being supposed, we consider small excitations of the vacuum at large distances. Two nontrivial modes prove to be possible, massive and massless ones.

Keywords: 8-spinor identity, Higgs-like potential, soliton solutions, vacuum excitations.
PACS: 03.50.-z, 03.65.-w, 03.65.Pm.

BRIOSCHI IDENTITY AND TOPOLOGICAL SOLITONS

The Skyrme's idea to describe baryons as topological solitons [1] proved to be fruitful in nuclear physics for modeling the internal structure of hadrons [2, 3] and light nuclei [4, 5]. In the Skyrme Model the topological charge Q = deg ($S^3 \to S^3$) is interpreted as the baryon number B and serves as the generator of the homotopy group $\pi_3(S^3) = Z$.

The similar idea to describe leptons as topological solitons was announced by L.D. Faddeev [6]. In the Faddeev Model the Hopf invariant Q_H is interpreted as the lepton number L and serves as the generator of the homotopy group $\pi_3(S^2) = Z$ .The unification of these two approaches was suggested in [7], hadrons and leptons being considered as two possible phases of the effective 8-spinor field model.

The basic idea was to take into account the existence of the special 8-spinors identity discovered by the Italian geometer Brioschi [8]:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2, \tag{1}$$

where the following quadratic spinor quantities are introduced:

$$s = \overline{\Psi}\Psi, \quad p = i\overline{\Psi}\gamma_5\Psi, \quad \vec{v} = \overline{\Psi}\vec{\lambda}\Psi,$$

$$\vec{a} = i\overline{\Psi}\gamma_5\vec{\lambda}\Psi, \quad j_\mu = \overline{\Psi}\gamma_\mu\Psi, \quad \tilde{j}_\mu = \overline{\Psi}\gamma_\mu\gamma_5\Psi,$$

with $\overline{\Psi} = \Psi^+\gamma_0$ and $\vec{\lambda}$ standing for Pauli matrices in the flavor (isotopic) space. Here the diagonal (Weyl) representation for $\gamma_5 = \gamma_5^+$ is used and μ = 0, 1, 2, 3, designate the unitary Dirac matrices acting on Minkowski spinor indices.

If one defines 8-spinors as columns:

$$\Psi = \text{col }(\psi_1, \psi_2), \; \psi_i = \text{col }(\varphi_i, \chi_i), \; i = 1, 2,$$

with φ_i and χ_i being 2-spinors, then one easily finds that the following identity holds:

$$2j_\mu j^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2 + \Delta^2, \quad (2)$$

showing the time-like character of the 4-vector j_μ where the denotation is introduced:

$$\Delta^2 = 8\left[(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) - |\varphi_1^+\varphi_2|^2 + (\chi_1^+\chi_1)(\chi_2^+\chi_2) - |\chi_1^+\chi_2|^2\right] \geq 0.$$

The structure of the identity Eq. 2 leads to the natural conclusion that the Higgs potential V in the effective spinor field model can be represented as the function of $j_\mu j^\mu$:

$$V = \frac{\sigma^2}{8}(j_\mu j^\mu - \varkappa_0^2)^2, \quad (3)$$

with σ and $\varkappa_0$ being some constant parameters.

If one searchs for localized soliton-like configurations in the model, one finds the natural boundary condition at space infinity:

$$\lim_{|\vec{x}|\to\infty} j_\mu j^\mu = \varkappa_0^2. \quad (4)$$

As follows from the identity Eq. 2, the condition Eq. 4 determines the fixed (vacuum) point on the surface S^8. Using Eq. 4 and the well-known property of homotopic groups of spheres: $\pi_3(S^n) = 0$ for $n \geq 4$, one concludes that the two phases with nontrivial topological charges may exist in the model Eq.3. The first one corresponds to the choice $\pi_3(S^3) = Z$ (Skyrme Model) and the second one coresponds to the choice $\pi_3(S^2) = Z$ (Faddeev Model).

For example, if the vacuum state Ψ_0 defines $s(\Psi_0) \neq 0$, then the configurations characterized by the chiral invariant $s^2 + \vec{a}^2$ determining sphere S^3 as the field manifold are possible, that corresponds to Skyrme Model phase.

On the contrary, if only $v_3(\Psi_0) \neq 0$, then the $SO(3)$ invariant $\vec{v}^2$ determines the S^2 field manifold, that corresponds to Faddeev Model phase.

EFFECTIVE NONLINEAR 8-SPINOR FIELD MODEL

In view of these topological arguments, using the analogy with Skyrme (or Faddeev) Model, we suggested in [7] the following Lagrangian density for the effective 8-spinor field model:

$$L = \frac{1}{2\lambda^2}\overline{\partial_\mu\Psi}\gamma^\nu j_\nu \partial^\mu\Psi + \frac{\varepsilon^2}{4} f_{\mu\nu} f^{\mu\nu} - V, \quad (5)$$

where $f_{\mu\nu}$ stands for the anti-symmetric tensor of Faddeev - Skyrme type:

$$f_{\mu\nu} = (\overline{\Psi}\gamma^\alpha \partial_{[\mu}\Psi)(\partial_{\nu]}\overline{\Psi}\gamma_\alpha\Psi),$$

with λ and ε being constant parameters of the model. It should be stressed that the first term in Eq. 5 generalizes the sigma-model term in Skyrme Model and includes the projector $P = \gamma^0\gamma^\nu j_\nu$ on the positive energy states. The second term in Eq. 5 gives the generalization of Skyrme (or Faddeev) term.

We intend now to study the behavior of possible topological soliton solutions in the model Eq. 5 at large distances that corresponds to small excitations of the vacuum:

$$\Psi = \Psi_0 + \xi, \quad \xi \to 0 \text{ at } |\vec{x}| \to \infty. \tag{6}$$

Inserting Eq. 6 into Eq. 5, one gets the second variation of the Lagrangian density:

$$\delta^2 L = \frac{1}{\lambda^2}\partial_\mu \xi^+ P_0 \partial^\mu \xi - \frac{\sigma^2}{4}(\delta j^2)^2, \tag{7}$$

where the following denotations were used:

$$\delta j^2 = 2 j_\mu^{(0)}(\bar{\xi}\gamma^\mu \Psi_0 + \overline{\Psi_0}\gamma^\mu \xi) \equiv \eta, \quad P_0 = \gamma^0 \gamma^\nu j_\nu^{(0)}, \tag{8}$$

with $j_\nu^{(0)}$ standing for the vacuum value of the current j_ν.

From Eq. 7 and Eq. 8 one easily derives the equations of motion for the perturbation ξ:

$$\frac{1}{\lambda^2}\partial_\mu P_0 \partial^\mu \xi + \sigma^2 \eta P_0 \xi = 0. \tag{9}$$

If one inserts into Eq. 9 $\xi = k\Psi_0$, $k^* = k$, and takes into account Eq. 8 and the boundary value Eq. 4, one finds that the scalar function $\eta = 4k\varkappa_0^2$ satisfies the well-known Klein – Gordon equation

$$(\partial_\mu \partial^\mu + M_0^2)k = 0 \tag{10}$$

with the mass term given by $M_0^2 = 4\lambda^2\sigma^2\varkappa_0^2$.

On the contrary, if one considers imaginary $k^* = -k$, $\xi = k\Psi_0$, then from Eq. 8 one derives $\eta = 0$ and the wave equation

$$\partial_\mu \partial^\mu k = 0, \tag{11}$$

that corresponds to the massless excitations in our model.

Thus, we conclude that our model admits two types of vacuum excitations, massive and massless ones. The evident interpretation of this result can be obtained if one considers the soliton-like excitation ξ as the wave function in the special stochastic representation of Quantum Mechanics [8].

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Scales and Explanation of Physical Effects

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Abstract. The approach of the relational statistical modeling the properties of the physical space and time is applied to consider some complicated phenomena of the modern cosmology. The main attention is paid to consideration of the Dark Matter (DM) problem. In contrast to the traditional view the explanation of the effects associated with DM can be made by changing the basic apparatus of the theory of gravitation in the framework of the relational statistical space-time concept. This is possible due to the main relationships of the concept in which the known physical equations are not postulated but are deduced from the basic relationships for determination of the space and time. Thus DM is treated as a fictitious mass which must be prescribed to the equations for the cosmological scales.

Keywords: relational statistical model of space-time, dark matter

PACS: 04.20CV, 04.25.Nx, 04.50.Kd.

INTRODUCTION

Challenges in the modern physics related to unexplained phenomena of Dark Matter (DM) and Dark Energy (DE) require experimental and theoretical investigations. The traditional view on the DM proposed the numerous attempts to explain this phenomenon in the framework of the classical gravitation theory that leads to search for the hidden matter. The unsuccessful attempts to find "particles of DM" (WIMP etc) is a one of the reasons to change the theory.

There are some approaches, for example semi-empirical theories, in particular, Modified Newtonian Dynamics (MOND) [1], in which corrections in the classical formulae are inserted. MOND introduces a constant value with the dimensions of an acceleration, and asserts that standard Newtonian dynamics is a good approximation only for accelerations that are much larger than this value. There are also contemporary attempts to change the physical and mathematical apparatus of General Relativity (GR). For instance in [2-3] isotropic cosmology built in the Riemann-Cartan space-time leads to solution of problems of DM and DE.

In contrast our approach tries to deduce the mentioned physical consequences from the basic equations for space and time. The relational statistical space-time concept allows us to describe some physical effects, in particular, gravitation without introduction of the notion of field, see [4, 5]. In the present paper the important conclusions of this approach are discussed. The Mach principal underlies the theory because space and time are treated, in fact, as statistical cosmological values. So the known relationships between microscopic and macroscopic level of description are deduced from the relations of the probability theory. Connection of the gravitation and the electromagnetism are also discussed. This link is derived not in terms of the field theory but on the basis of the relational concept.

In the present paper the problem of DM is only considered and DM is treated as a fictitious value. It is due to the connection between mass (matter) and distance which is one of the main postulates of our concept. Namely, in this concept the distance can be measured in units of the mass with introducing the statistical model of the rods. The structure of the space is related to the configuration of particles in the Universe, so different distributions determine on the gravitation relationships.

STATISTICAL RELATIONAL APPROACH

The statistical relational space-time concept implies constructing the physical theory based on the generalized models of fundamental instruments for measuring space and time. In so doing one can introduce the models of physical space and time and deduce the appropriate equations. We present general formulae from [6-8].

We postulate an equation for a time increment $d\tau$ in which the set of all particles of the system under consideration simulates these properties. An increment of time is obtained by infinitesimal increments of the coordinates of the particles (determined by the idealized photo-camera in the given point):

$$d\tau^2 = \frac{a^2}{N}\sum_{i=1}^{N}(dr_i - \frac{1}{N}\sum_{j=1}^{N} dr_j)^2. \qquad (1)$$

Here *a=1/c*, where *c* is the speed of light (it is proved in [6]).

«Discrete scale ruler» and main equations of the relational space can be deduced in the following way. A model of the fundamental device is founded by means of the uniform distributed discrete medium of particles. A process of measurement consists in the correspondence between the body and a system of particles of the discrete medium. Formalization of the measurement process introduces a model of the relational statistical discrete space. We cannot measure a distance less than "one particle". Here is a minimum length

$$r_e = bm_e, \qquad (2)$$

where m_e is the mass of the particle (atom) and *b* the factor determined from applying the concept to quantum effects. One can obtain with the use Eq. 2, see [9]

$$b = h/(m_e^2 c), \qquad (3)$$

where *h* is the Planck constant. The distance can be expressed through the distribution (configuration) of masses of the system under consideration which can be represented by the following average:

$$dr = \frac{b}{N}\sum_{i=1}^{N} dm_i. \qquad (4)$$

One of the essential properties of this approach implies that space and time and consequently the dynamics of the system (in particular the Universe) are determined and expressed directly through the distribution of the moving set of the particles (atoms) of the all system. The time value is expressed from Eq. 1 through the spatial units with the appropriate recalculation using the factor *a*. The space unit in turn can be expressed from Eq. 4 through the mass units with the appropriate recalculation using the factor *b*. The last fact plays the crucial role in the explanation of DM effects, because the distances and masses can in fact be compared. We emphasize that Eqs.1-4 allow to deduce physical equations from mathematical relations.

THE PROBLEM OF DM IN THE FRAMEWORK OF THE RELATIONAL STATISTICAL CONCEPT

The concept of the relational statistical space-time in the cosmological scales allows us to understand gravitational effects. Time and space according to the model formulae (1) and (4) correspond to the measurements of the physical clocks and rules respectively. But this is true with the limiting accuracy for the uniform distributions of masses in the system under consideration and for the uniform motion of all particles. For the situation of “the mass clots”, i.e. of the body with the nonunity mass (if the local distribution of masses differs from the average) there are distinction from the readings of the appropriate fundamental devices and this can be treated as the manifestation of the gravitation. Such an approach can reproduce some known consequences of General Relativity, e.g. the effect of the curve of the light ray and the gravitation lensing. Moreover the concept is in fact a realization of the Mach principal.

The notion of DM can be reduced to the manifestation of the statistical properties with essential nonuniformity of the matter at the cosmological scales. The formula for the ordinary Newtonian gravitational potential is derived from the statistical relations when particles in the Universe is uniformly distributed. Then the most part of particles is in the distance of *R* from the particle under consideration, where *R* is the radius of the Universe. The important assumption is that all physical relationships are based and derived from the mathematical relations for the space-time, so the proposed relational statistical space-time concept provides all equations including relationships for DM.

With the nonuniformity, e.g. the existence of the big galaxy, the uniformity is disturbed. In this case the effective distance to the numerous particles of the galaxy can be significantly smaller than *R*. Thus the gravitational potential would be increased. But as the expression for the potential is derived from the relation of two dimensionless sum (and this value is approximately constant), for “compensation” one can increase the mass with the same distance of the order of *R*. In this case one cannot suppose that DM exists, but the effects of DM can be explained by the generalization of the relational statistical concept.

The ordinary gravitational potential is obtained (see [8]) by comparing two dimensionless (taking into account the mentioned fact about measuring time in units of space and respectively space in mass units) sum by means of the probability theory

$$\sum_i \frac{m_e}{r_i} \sim \sum_i \frac{u_i^2}{c^2},$$

where $r_i \sim R$. The last equation is valid because for the uniform distribution of the matter in the world one can suppose that the most part of the effective matter is concentrated in the layer of the sphere at the scale of the order of R, thus one can use the sum of the stochastic values of the same order.

More correctly this equality can be rewritten as follows

$$\frac{1}{N}\sum_{i=2}^{N}\frac{(u-\frac{1}{N}\sum_{j=1}^{N}u_j)^2}{c^2} = \frac{1}{N}\sum_{i=2}^{N}\frac{m_e}{m_{1i}} + O(\frac{1}{\sqrt{N}}) \qquad (5)$$

where N is the so-called Eddington number (the number of nucleons in the Universe). Here the 1-st particle is considered and the distance from this particle to the other particles measured in terms of the appropriate masses are written.

From Eq. 5 using the appropriate dimension values one can derive the ordinary expression for Newton's gravitation potential

$$\phi_{гр.N} = -G\sum_{i=2}^{N}\frac{m_e}{r_{1i}},$$

where G is the gravitation constant.

Eq. 5 is a realization in the concept of the Mach principle and the statistical relationships applied for the global scales leads to reproducing some relationships between fundamental constants. In particular one can derive for constant b from Eq. 3

$$b = (\sqrt{N}G)/c^2,$$

For the large masses of the astrophysical objects the linear relation between distance and mass ("connection" of distance and mass) results in changing the dynamical equations. For compensation of a large number of particles at the distance smaller than the radius of the Universe (a galaxy etc) $r_i < R$ it needs to prescribe the additional mass m_i' in such a manner that

$$b\sum_i \frac{m_i + m_i'}{r_i'} \sim \sum_i \frac{u_i^2}{c^2},$$

where $r_i' \sim R$. In this expression the mentioned idea of the relational statistical model of the space is realized, that the distance can be measured in fact in the mass units. This can be associated with the suggestion of DM theory in which each galaxy contains a halo of an as yet unidentified type of matter that provides an overall mass distribution different from the observed distribution of normal matter.

In more detail in a case of the large concentration of particles at distances smaller than the radius the sum in the right size will be greater. With the aim to conserve the previous form in the right hand side one can introduce the fictitious mass and the previous value of the distance:

$$\phi_{ep.N} = -G\sum_{i=2}^{N} \frac{m_e}{Ar_{1i}} = -\frac{G}{A}\sum_{i=2}^{N} \frac{m_e + m_e^{'}}{r_{1i}}$$

Here $A = \sqrt{N}$. This can be made due to the mentioned "connection" of distance and mass. The constant A is as follows $R = Ar_e$ and this magnitude of this constant is estimated in [8].

In the case of large masses (for example the concentration of particles such as the galaxy) the uniformity of the distribution of distances is also destroyed. In other words, one can compensate changing r_{1i} to R by the appropriate increase of the mass Mm_e to $Mm_e + M'm_e$, where M is the number of particles in the galaxy under consideration and M' is the fictitious mass of DM. Let us consider this derivation.

$$\frac{1}{A_1 b}\sum_{i=2}^{N} \frac{m_e}{m_{1i}} = \frac{1}{A_1}\sum_{i=2}^{N} \frac{m_e}{r_{1i}} = \frac{1}{Ab}\sum_{i=2}^{N} \frac{Am_e}{A_1 m_{1i}} = \frac{1}{A}\sum_{i=2}^{N} \frac{Am_e}{A_1 r_{1i}} = \frac{1}{A}\sum_{i=2}^{N} \frac{m_e + m'}{r_{1i}}$$

Here $A_1 < A$, the part of the particles for which the radius is smaller than R can be considered in the real situation. Thus the generalization of theory of gravitation on the basis of the relational statistical space-time excludes the existence of DM.

In this short paper we try to show how the theory of space-time introduces the effective mass which can be treated as DM, the next possible step would be an analysis of the real distributions stellar and galaxy systems to consider the correspondence of the theoretical predictions with the experimental data.

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Stationary states of non-relativistic electron in magnetic-solenoid field: Classical orbits approach

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Abstract. The main task of this work is to evaluate the influence of a magnetic-solenoid field on stationary quantum states of a non-relativistic charged particle (electron). In the problem under consideration the classical orbits approach is associated to the two kinds of stationary quantum states: those which correspond to the classical trajectories which embrace the solenoid and those which do not.

Keywords: Aharonov-Bohm effect, Magnetic-solenoid field, Stationary States.
PACS: 03.65Ta

INTRODUCTION

It is known that the study of the Aharonov-Bohm effect was based on a mathematical treatment which consider the exact wave functions of an electron in the field of an infinitely long and infinitesimally thin solenoid [1]. Such functions allow one to analyze a nontrivial influence of the Aharonov-Bohm solenoid on scattering of free electron, which may give a new interpretation of eletromagnetic potentials in quantum theory. Physically it is clear, that in such a scattering, the electron is subjected to the action of the Aharonov-Bohm field for a short finite time. However, there exist a possibility to consider bound states of the electron on which it is affected by the Aharonov-Bohm field for the infinite time. Such bound states exist in the so-called magnetic-solenoid field, which is a superposition of the field of Aharonov-Bohm solenoid and a collinear constant and uniform magnetic field [2]. We believe that such bound states of an electron in the magnetic-solenoid field open new possibilities to the study of the Aharonov-Bohm effect. The functions which are solutions of the schrodinger equations, or in other words the stationary quantum states are constructed [3,6,8]. Moreover, the two kinds of stationary quantum states, those which correspond to the classical trajectories which embrace the solenoid and those which do not are pointed out. We consider the non-relativistic motion of an electron with charge $q = -e$, $e > 0$, and mass M in the magnetic-solenoid field $\mathbf{B} = (B_x, B_y, B_z)$,

$$B_x = B_y = 0, \tag{1}$$

$$B_z = B + \Phi\delta(x)\delta(y) = B + \frac{\Phi}{\pi r}\delta(r), \tag{2}$$

which is a collinear superposition of a constant uniform magnetic field B directed along the axis z $(B > 0)$ and the Aharonov-Bohm field (field of an infinitely long and infinitesimally thin solenoid) with a finite constant internal magnetic flux Φ. We use Cartesian coordinates x, y, z, as well as cylindrical coordinates r, ϕ, such that

$$x = r\cos\varphi, \quad y = r\sin\varphi, \tag{3}$$

and $r^2 = x^2 + y^2$. The field (1, 2) can describe by the vector potential $\mathbf{A} = (A_x, A_y, A_z)$,

$$A_z = 0, \tag{4}$$

$$A_x = -y\left(\frac{\Phi}{2\pi r^2} + \frac{B}{2}\right), \tag{5}$$

$$A_y = x\left(\frac{\Phi}{2\pi r^2} + \frac{B}{2}\right). \tag{6}$$

Classical motion of the electron in the magnetic-solenoid field is governed by the Hamiltonian

$$H = \mathbf{P}/2M, \qquad \mathbf{P} = \mathbf{p} - \frac{q}{c}\mathbf{A}, \tag{7}$$

where p and P are the generalized and kinetic momentum, respectively. Trajectories that do not intersect the solenoid have the form:

$$x = x_0 + R\cos\Psi \qquad (8)$$

$$y = y_0 + R\sin\Psi \qquad (9)$$

$$z = \frac{p_z}{M}t + z_0 \qquad (10)$$

$$\Psi = \omega t + \Psi_0, \quad \omega = \frac{eB}{Mc} \qquad (11)$$

where $x_0, y_0, z_0, p_z, R,$ and are integration constants. The expressions (8, 9, 10, 11) imply

$$(x - x_0)^2 + (y - y_0)^2 = R^2, \qquad (12)$$

$$x_0 = R_c\cos\alpha, \quad y_0 = R_c\sin\alpha, \qquad (13)$$

$$r^2 = R^2 + R_c^2 + 2RR_c\cos(\Psi - \alpha), \qquad (14)$$

$$R_c = \sqrt{x_0^2 + y_0^2}. \qquad (15)$$

The projection of particle trajectories on the xy-plane are circles. Particle images on the xy-plane are rotating with the cyclotron frequency ω. For an observer which is placed near the solenoid with z > *0*, the rotation is anticlockwise. The particle has a constant velocity $\frac{p_z}{M}$ along the axis z. Since the electron freely propagates on the z axis, only motion in the perpendicular plane $z = 0$ is nontrivial; this will be examined below. denoting by r_{max} the maximal possible moving off and by r_{min} the minimal possible moving off of the particle from the z-axis, we obtain from (12,13,14,15) $r_{max} = R + R_c, r_{min} = |R - R_c|$ .It follows from (8,9,10,11) that

$$P_x = -M\omega R\sin\Psi = -M\omega(y - y_0) \qquad (16)$$

$$P_y = M\omega R\cos\Psi = M\omega(x - x_0) \qquad (17)$$

$$\mathbf{P}_\perp^2 = P_x^2 + P_y^2 = (M\omega R)^2 \qquad (18)$$

The energy E of the particle rotation reads $E = \mathbf{P}^2/2M = M(\omega R)^2/2.$.
By the help of (16,17,18), it is also convenient to introduce the conserved quantity in terms of L_z as

$$\nu = L_z + \frac{e\Phi}{2c\pi} = \frac{M\omega}{2}\left(R^2 - R_0^2\right) \ , \qquad (19)$$

Already in classical theory, it is convenient, to introduce dimensionless complex quantities a_1 and a_2 (containing $\hbar$) as follows:

$$a_1 = \frac{-iP_x - P_y}{\sqrt{2\hbar M\omega}} = \sqrt{\frac{M\omega}{2\hbar}} R e^{-i\Psi}, \tag{20}$$

$$a_2 = \frac{M\omega(x+iy) + iP_x - P_y}{\sqrt{2\hbar M\omega}} = \sqrt{\frac{M\omega}{2\hbar}} R_c e^{i\alpha}. \tag{21}$$

One can see that $a_1 e^{i\Psi}$ and a_2 are complex integrals of motion. One can write some physical quantities in terms of a_1 and a_2

$$R^2 = \frac{2\hbar}{M\omega} a_1^* a_1, \tag{22}$$

$$R_c^2 = \frac{2\hbar}{M\omega} a_2^* a_2, \tag{23}$$

$$r^2 = \frac{2\hbar}{M\omega}(a_2 - a_1^*)(a_2^* - a_1), \tag{24}$$

$$E = \omega\hbar a_1^* a_1, \tag{25}$$

$$L_z = \hbar(a_1^* a_1 - a_2^* a_2) - \frac{e\Phi}{2\pi c}. \tag{26}$$

STATIONARY QUANTUM STATES

The quantum behavior of the electron in the field (1, 2) is determined by the Schrodinger equation with the Hamiltonian

$$\hat{H} = \hat{H}_\perp + \hat{p}_z^2/2M, \quad \hat{H}_\perp = (\hat{P}_x^2 + \hat{P}_y^2)/2M, \tag{27}$$

$$\hat{P}_x = \hat{p}_x + \frac{e}{c}A_x, \quad \hat{P}_y = \hat{p}_y + \frac{e}{c}A_y, \tag{28}$$

$$\hat{p}_x = -i\hbar\partial_x, \quad \hat{p}_y = -i\hbar\partial_y, \quad \hat{p}_z = -i\hbar\partial_z, \tag{29}$$

where $\hat{H}_\perp$ determines the nontrivial behavior on the *xy*-plane. It is convenient to presente magnetic flux Φ in equation (2) as $\Phi = (l_0 + \mu)\Phi_0$, where l_0 is integer, and $0 \le \mu < 1$ and $\Phi_0 2\pi c\hbar/e$ is Dirac's fundamental unit of magnetic flux. Mantissa of the magnetic flux μ determines, in fact, all the quantum effects due to the presence of the Aharonov-Bohm field. The corresponding radial functions were taken regular at $r = 0$, they correspond to a most natural self-adjoint extension (with a domain $D_{\hat{H}_\perp}$) of the

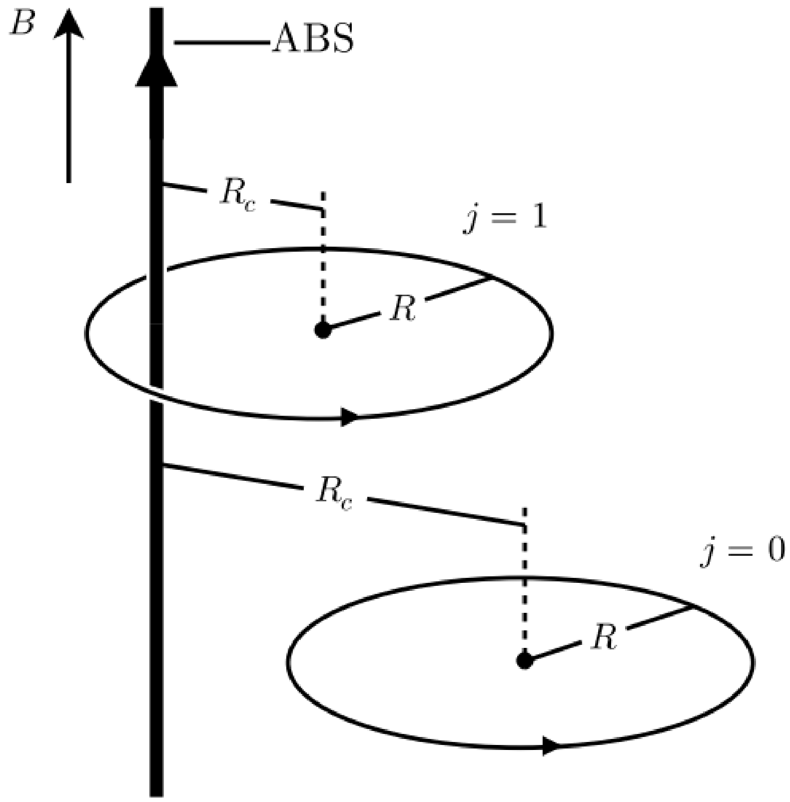

FIGURE 1. Two types of trajectories

differential symmetric operator $\hat{H}_\perp$. Operator $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$ is self-adjoint on $D_{\hat{H}_\perp}$ and commutes with the self-adjoint Hamiltonian $\hat{H}_\perp$. It is convenient to evaluate two types (*j = 0* and *j = 1*, which are associated to *v > 0* and *v < 0*, respectively) of common eigenfunctions of both operators [4,5]

$$\hat{H}_\perp \Psi^{(j)}_{n_1,n_2}(t,r,\varphi) = \mathcal{E}_{n_1} \Psi^{(j)}_{n_1,n_2}(t,r,\varphi), \tag{30}$$

being

$$\mathcal{E}_{n_1} = \hbar\omega(n_1 + 1/2), \tag{31}$$

$$\hat{L}_z \Psi^{(j)}_{n_1,n_2}(t,r,\varphi) = L_z \Psi^{(j)}_{n_1,n_2}(t,r,\varphi), \tag{32}$$

where

$$L_z = \hbar(l - l_0) \tag{33}$$

The eigenfunctions have the form

$$\Psi^{(j)}_{n_1,n_2}(t,r,\varphi) = e^{(-\frac{i}{\hbar}\mathcal{E}_{n_1}t)} \Psi^{(j)}_{n_1,n_2}(\varphi,\rho), \tag{34}$$

where

$$\rho = \frac{eBr^2}{2c\hbar}, \quad j = 0, 1. \tag{35}$$

Still, it is convenient to present the two functions

$$\Phi^{(0)}_{n_1,n_2}(\varphi,\rho) = \aleph e^{[i(l-l_0)\varphi]} I_{n_1,n_2}(\rho), \tag{36}$$

where $n_1 = m, n_2 = m - l - \mu, -\infty < l \leq -1,$

$$\Phi^{(1)}_{n_1,n_2}(\varphi, \rho) = \aleph e^{[i(l-l_0)\rho + \pi l]} I_{n_1,n_2}(\rho), \tag{37}$$

where $n_1 = m + l + \mu, n_2 = m, 0 \leq l \leq +\infty.$ Here *l, m (m ≥ 0)* are two integers, $I_{n,m}(\rho)$ are Laguerre functions that are related to the Laguerre polynomials $L^{\alpha}_{m}(\rho)$ [7] as follows

$$I_{m+\alpha,m}(\rho) = \sqrt{\frac{\Gamma(m+1)}{\Gamma(m+\alpha+1)}} e^{(-\rho/2)} \rho^{\alpha/2} L^{\alpha}_{m}(\rho), \tag{38}$$

being

$$L^{\alpha}_{m}(\rho) = \frac{1}{m!} e^{\rho} \rho^{-\alpha} \frac{d^m}{d\rho^m} e^{-\rho} \rho^{m+\alpha}. \tag{39}$$

and ℵ is normalization constant. These functions form a complete orthogonalized set on $D_{\hat{H}_\perp}$. It is useful to define self-adjoint operators $\hat{R}^2$ and R^2_c by analogy with the corresponding classical relations:

$$\hat{R}^2 = \frac{2\hat{H}_\perp}{M\omega^2} \tag{40}$$

$$\frac{2}{M\omega}[\hat{L}_z + (l_o + \mu)\hbar] = \hat{R}^2 - \hat{R}^2_c. \tag{41}$$

In the semiclassical limit the sign of the mean value of the operator $\hat{R}^2 - \hat{R}^2_c$ allows one to interpret the corresponding states as particle trajectories that embrace and do not embrace the solenoid "**Figure 1**". Namely, as follows from (32) and (41) an orbit embraces the solenoid for *l ≥ 0 (j = 1),* and do not for *l ≤ −1 (j = 0). If μ ≠ 0,* energy levels states (34) with *l ≥ 0* are shifted with respect to the Landau levels by μω, while energy levels of states (34) *l ≤ −1* remain coincide with the Landau levels.

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A Few Consequences of the Isotopic Field-Charge Spin Conservation

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Abstract. Let us start from two preliminary assumptions (Darvas, 2011): Although *mass of gravity* and *mass of inertia* are *equivalent* quantities in their measured values, they are *qualitatively not identical* physical entities. We take into consideration the difference between *mass of gravity* and *mass of inertia* in our equations. Although they are *equivalent* quantities in their measured values, they are *qualitatively not identical* physical entities. Then we extend this '*equivalence is not identity' principle* to sources of further fundamental interaction fields, other than gravity (like electric charge, weak charges, colours). We call the two qualitatively different physical quantities "isotopic field-charges". Provided that isotopic field charges can be distinguished in their physical behaviour, these qualitatively different *entities* interact with each other. These two assumptions allow an alternative interpretation of our physical experience. Based on them we demonstrated the existence of an invariance between the two isotopic forms of the field-charges, first in mathematical terms (Darvas 2009) then the corresponding physical conservation law (Darvas. 2011). Based on these results, we formulate certain consequences, in our view, on the physical structure of matter. In this course, the emphasis will be placed on the prediction of a family of intermediate bosons. The proposed model clusters observations in another way than usual, it extends the Standard Model. Unlike existing alternative theories, e.g., the SUSY, which renders a new ("supersymmetric") brother to each particle, this model clusters the observed sources of fields in two-eggs twin pairs, regarding them as isotopic states of each other, and there is left "only" the twin brothers of the bosons mediating their interactions to be observed. The extended model covers gravitational, electroweak and strong interactions. In contrast to the SUSY, which renders fermion-boson pairs as new-born brothers to each other, the Isotopic Field-Charge Spin (IFCS) theory, expanded in (Darvas, 2011), renders fermion-fermion and boson-boson twins to each other. This model does not assume new fermions; the twin brothers of fermions originate in splitting the existing ones. Fermions split as a result of a newly interpreted property. The assumption is mathematically based (Darvas, 2009) on an invariance of interactions under rotation of the isotopic field charges' spin (a property that distinguishes the

field charge twins from each other) in a still hypothetical, kinetic gauge field, that means, on the conservation of the isotopic field charge spin.

Keywords: Kinetic field, Equivalence principle, General Theory of Relativity, High energy physics, Symmetry, Invariance, *Isotopic field-charges* for the *Gravitational interaction.* Conservation, Isotopic field-charge spin, Symmetry breaking.

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INTERACTION BETWEEN THE ISOTOPIC FIELD CHARGES

Take a measure on an object! You will have no experience that you found it in one or the opposite isotopic state. Would you observe a single particle, it were either in one or in the other IFCS state. We can call the two states as potential and kinetic, scalar and vector, or bound and free states. However, your measurement records a mixture of the two states. Nevertheless, you do not observe the individual IFCS states. Your observation suggests that they behave as being in both states, each measured object can occupy both a potential (bound) and a kinetic (free) IFCS state. In the lack of experience to catch a particle in one or the other stable state, we have good reason to assume that they permanently change their states. (Randomly or with a stable frequency, they may probably follow a similar mechanism like quarks do during their colour change via gluon exchange in the Standard Model).

Let us consider a model of a doublet, when a particle can be in a potential state (V) and in a kinetic state (T). According to its actual state it has potential or kinetic energy respectively. According to our observation all particles possess both. We can interpret the phenomenon in the following way: In a *probabilistic model* we can consider that the wave function of the given particle may be in a potential state with amplitude ψ_V, or in a kinetic state with amplitude ψ_T. We detect a probabilistic mixture in a measurement. In a large set of particles (e.g., in the case of a massive body consisting of many particles) the probabilities reach a stable proportion and we observe stabilised measurable potential and kinetic energies in a given reference frame. A *harmonic oscillator model* presumes the permanent change of a single particle between its two isotopic field charge states. A particle *in a potential state* plays the role of the *source of a scalar field*. Therefore a potential isotopic field charge (we denote by $\daleth_V$) is a scalar quantity. A particle *in a kinetic state* serves as a *current source of a vector field*. So a kinetic isotopic field charge (we denote by $\daleth_T$) plays roles in three vector components according to three, directed, independent components of a field charge current. An important consequence of the switch between the two IFC states is that the isotopic field charges must commute between a scalar and three components of a vector quantity, according to the velocity components of the kinetic state in the given reference frame.

Equivalence does not mean identity

The equivalence principle is one of the main pillars of the general theory of relativity (GTR) [2]-[11]. It states the equivalence of the gravitational and inertial masses. Let's consider the mass of gravity and the mass of inertia as two different properties of matter. For the same massive object can behave once as a source of gravity, then as a measure of inertia, we iinterpret them as two isotopic states of the same property, called mass of the object.

Identical objects cannot be equivalent. Only *qualitatively different* objects can be compared to conclude a *quantitative equivalence* between them. Equivalence always presumes the existence of at least one property, in which the compared objects differ. (Isotopic spin is a good example how to avoid ambivalence.)

As much as the mass is the field charge of the gravitational field, we call its two isotopic states as *isotopic field charges* for the *gravitational interaction*. The gravitational mass is associated with the (scalar) potential part of that interaction, while the inertial mass with the kinetic part. In GTR the latter is attributed to the momentum densities, while the former is associated with the gravitational field energy. They are separated within the stress-energy tensor ($T_{\mu\nu}$), but according to the general relativity principle they can be transformed into each other; – we should add, at least in their quantitatively equivalent values. GTR does not make any statement about the qualitative transformation of the two kinds of masses into each other. This was a reason to identify them. The need for a qualitative transformation simply has not emerged. Nevertheless, we show that it cannot be avoided. So, we introduce distinction between masses of gravity and inertia in our equations. (In a similar way, the electric charge – i.e., the source of the electromagnetic field – is the field charge of the electromagnetic interaction; flavour and lepton charge – are the sources of the weak field; the colour charge – i.e., the source of the strong field – is the field charge of the strong interaction.) The sources – field charges – are assumed to be realised in the matter field, while they serve as sources for gauge fields. Are they really the same, or can one distinguish the two agents? The mass of gravity and the mass of inertia are considered as two equivalent quantity *isotopic states* of the field charge of the gravitational field. They represent two different qualities. Their concepts express two properties of matter, whose existence originates in different experiences. Physics established quantitative relations between them (i.e., equal values), however this fact does not vanish their qualitative difference. We argue that we have all reason to make distinction between them in our theories.

When we introduce the two isotopic filed charges in our equations, they destroy certain symmetries of those equations. This contradicts to our experience. Therefore, there must be an invariance that compensates and restores the spoiled symmetry. To avoid the contradiction between experience and theory, we assume that the two kinds of charges of the gravitational field should be transformed into each other by a gauge transformation. Such a gauge transformation should involve the existence of a conserved property that we define in the following way.

- Since the required transformation affects the *isotopic state*s of the individual *field-charges* (we mark it with ד ['dalet' the fourth letter of the Hebrew alphabet]), this transformation must be performed in a special gauge field; and since these states can occupy two positions in that gauge field, it must be a *spin-like property*, therefore, we call this property as *Isotopic Field-Charge Spin (IFCS)* and denote it by *Δ*, and we refer to the invariance transformation what we are seeking for as *isotopic field-charge gauge transformation*. This assumption assumes the existence of a local gauge field, in which the isotopic field-charge spin can rotate and occupy two states and concludes a conserved (non-Abelian) current and a corresponding class of *SU*(2) type invariances.
- For the same object can behave, e.g., in the gravitational field, once as the source of a gravitational force, and in another frame of reference as a source of a (kinetic) inertial force (cf., covariance principle), they must be able to get transformed into each other. Non-Abelian character and arbitrariness involve that the orientation of the isotopic field-charge spin is of no physical significance. If we determine the proper form of this invariance transformation, it will counteract the loss of symmetry between the two kinds of field-charges, and bring our equations in compliance with the experimental observations.
- The required invariance shows certain formal similarities to YM-type invariances [24]-[25]. However, it must differ from them in at least two features. Once, the concerned physical property, namely the isotopic field charge (IFC, ד), is a quite different physical property than the isotopic states of nucleons. Secondly, the gauge field – and consequently the gauge transformation that rotates the isotopic field charge spin (IFCS, *Δ*) in this gauge field – are quite different from the isotopic gauge field derived for the isotopic spin transformation. (For specification, see later sections.)

The existence of such an invariance transformation provided a symmetry, and consequently a conservation law, with the conservation of the introduced new property (*Δ*) of the field-charges. The conservation of isotopic field-charge spin is identical with the requirement of invariance of all interactions under isotopic field-charge spin rotation (in the gauge field where it is interpreted). Accordingly, all physical interactions should be invariant under a transformation in a specific gauge field, more precisely, under a rotation of the property, called isotopic field-charge spin (*Δ*). [16]-[18] proved that invariance transformation.

ISOTOPIC FIELD CHARGES IN THE GRAVITATIONAL FIELD

As a consequence of the distinction between m_V and m_T, as well as the association of the energy content with the mass m_V and the components of the momentum with m_T, we lose also the symmetry of the $T_{\mu\nu}$ energy-momentum tensor. To retain symmetry in Einstein's field equations we must require again the invariant transformation of m_V and m_T into each other in an appropriate gauge field. We refer to Mills [25] who foresaw the possible generalisation of YM type gauge invariance in general relativity "in close analogy with the curvature tensor". If we consider the energy-momentum tensor (in which both isotopic states of mass appear) as the source of the gravitational field, then – in the usual way – a scalar and a vector potential can be separated. (A hypothetic vector potential is justified by a non-static effect, e.g., acceleration, in the field.) Although, unlike QED, there is no analogy with the meaning of a vector potential of the electromagnetic field, the consideration of the kinetic (inertial) mass as an individual physical property against the gravitational mass may lend certain meaning to a gravitational vector potential. We can explain this so, that m_4 in T_{44} does not compose a fourth component of a four-vector in the classical theory of gravitation where there is a single scalar mass, while if we consider now $m_4 = m_V$, the three components of the kinetic mass m_T can compose a three-vector, however T_{i4} will not form a four vector either.

To maintain the Lorentz invariance of our physical equations in the gravitational field, we must demand to restore the invariance of $\begin{pmatrix} \vec{m}_T \\ m_V \end{pmatrix}$ under an *additional transformation* that should *counteract the loss of symmetry caused by the introduction of two isotopic states of mass*. We discuss that transformation in section 2. Further, in the case of gravitation the relation of the scalar and the vector fields are not linear even if we have not made distinction between the potential and kinetic masses. The non-linearity is coded in the relation of the tensors [26] at the left side of the Einstein equation (in units $c = 1$),

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

or $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ where the Einstein tensor is defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ whose covariant derivative must vanish.

Since our $T_{\mu\nu}$ tensor on the right side has already lost its symmetry, we can take $\Lambda g_{\mu\nu}$ into account within a modified $T'_{\mu\nu}$ – handling the gravitational and kinetic masses in it together with the dark energy – and we get the following formally symmetric equation:

$$G_{\mu\nu} = 8\pi G T'_{\mu\nu} .$$

(The disadvantage of this apparently quasi-symmetric form is that the metric tensor $g_{\mu\nu}$ appears in the expressions at both sides of the equation.) It is only our enigmatic hope that the asymmetry hidden inside $T'_{\mu\nu}$ will be restored with the conservation of the IFCS for the isotopic gravitational field charges together with the dark energy. Nevertheless, even if the latter fails, the symmetry of the energy-momentum tensor can be saved by the invariant gauge transformation of the IFCS. The most important analogy is between the behaviour of the potential and the kinetic field charges of the individual fields that makes probable to conjecture that a unique transformation will assure their invariance.[1] (See in details in the next sections.)

THE ISOTOPIC FIELD-CHARGE SPIN (*Δ*) CONSERVATION

Distortion of symmetry of our equations[2] is not in accordance with experience. It can be restored by proving that there exists an invariance between the twin brothers of the field charges (sources of the fields) split according to the introduced new property (*Δ*). Invariance means that particles, disposed with these properties, can be exchanged. The "exchange rate" (gauge) depends on the velocity of the kinetic field charge compared to the respective matter field (i.e., to the scalar potential field charge in rest in that field). The validity of the assumption can be verified by demonstrating the existence of the gauge bosons that mediate the exchange. This invariance – as soon as proven – means a new symmetry principle of nature. This perspective is challenging!

We present below the main lines of the mathematical proof [18] of such invariance. The demonstration of the predicted gauge bosons is left to the experimental physicists working on the observation of decay products at high energy collisions.

Velocity dependent phenomena

We know certain phenomena in classical physics that depend on velocity in a given reference frame. As examples, there can be mentioned first the kinetic energy, then the Lorentz force, and the covariant effect of the Lorentz transformation. Descriptions of the mentioned phenomena handle the space-time co-ordinates as indirect variables. The Lorentz invariance depends only on the velocity difference between the compared systems. In general, kinetic quantities depend first on velocity in the chosen reference frame, and only indirectly, through $v = v(x_i, t)$ on the space-time variables. As [25] observed, "Hamilton's principle was first discovered in connection with mechanical systems, where the Lagrangian turns out to be the difference between the kinetic and potential energies, but the principle is easily extended to include velocity-dependent forces of certain types", including, e.g., the magnetic force on a moving, electrically charged particle.

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[1] Let we make a few remarks in addition to the conjecture of the "unique" transformation. As [27] stated, "In contrast to the symmetry or *invariance* requirement in STR, the principle in GTR is most often presented as strictly speaking a *covariance* requirement." Gauge theories behave like GTR, at least in this respect. General covariance "is not tied to any geometrical regularity of the underlying spacetime, but rather the form invariance (covariance) of laws under arbitrary smooth coordinate transformations" [27, p. 34]. Weyl [28] found that the more general geometry resulting from admitting local changes called gauges described not only gravity but also electromagnetism. He showed also that the conservation laws of Noether follow in two distinct ways in theories with local symmetries. This led to the Bianchi identities, which hold between the coupled equations of motion, and which are due to the local gauge invariance of action. Later [29] demonstrated that the conservation of the electric charge followed from the local gauge invariance in the same way as does energy-momentum conservation from co-ordinate invariance in GTR.

[2] According to Higgs [30]: "The idea that the apparently approximate nature of the internal symmetries of elementary-particle physics is the result of asymmetries in the stable solutions of exactly symmetric dynamical equations, rather than an indication of asymmetry in the equations themselves, is an attractive one." Please, compare this notice with Wigner's concern [31]!

It is not surprising that phenomena related solely to the kinetic part of the Hamiltonian (T) can be described in a velocity dependent, i.e., kinetic field $D_T = D[v(x_i, t)]$ where the dependence on the local co-ordinates is indirect. This does not disclose the possibility of localisation of the theory in space-time, however, it does not ensure it automatically. Local symmetry in a kinetic field means that the objects, fields or physical laws in question are invariant under a local transformation, namely under a set of continuously infinite number of separate transformations with an arbitrarily different one at every velocity in the given reference frame.

The isotopic field charge (IFC, ℸ) as a property can be identified in the case of the gravitational field with the properties of the masses of gravity and inertia respectively. The potential isotope of ℸ ($ℸ_V$) depends directly on space-time co-ordinates. The physical state of the kinetic isotope of ℸ ($ℸ_T$) depends primarily on the components of its velocity (and indirectly on its space-time co-ordinates). When we try to specify physical phenomena that distinguish kinetic behaviour of objects from their behaviour in a field caused by another, potential source (i.e., $ℸ_V$) we should make attempt to seek for a description in a velocity dependent field.

Mathematical proof of the conservation of *Δ*

For the sake of the description of the mentioned distinction, we introduce a gauge field D_μ, that depends primarily on velocity. We derived a set of conserved currents in such a field [18]. The mathematical treatment is as much general as possible, while we made a specification. Namely, Noether's second theorem allows the dependence of the concerned fields (on which the Lagrangian depends) on any, general co-ordinate. Certain physical theories restrict themselves on the four space-time co-ordinates as dependent variables. We discuss fields that depend on co-ordinates in the velocity four-space, (and handle the space-time co-ordinates as indirect variables).

For the effects of a general non-Abelian group on the local gauge invariance are to be described, we refer to the [25] review paper. We partially use the methods of his description of YM type gauge fields. We introduce a new type of localised gauge field that does not coincide with the isotopic spin's YM field, marked by **B** in [24] and [25]; this field, marked by **D,** is *per defintionem* different from the YM field.[3] In our discussion, the **D** gauge field, introduced below, depends directly on the velocity-space coordinates, while the matter field depends directly on the four dimensional space-time co-ordinates. In other words, this means that although we primarily use coordinates of the velocity-space, our derivations are indirect and include derivatives with respect to the space-time co-ordinates (cf., the introduction of the relativistic λ_μ^ν tensor below) and play important role in our conclusions. This is an expression of the facts that we observe the physical events (occurring even in the velocity space) with respect to the 4D space-time, on the one

[3] Although we use the letter "D" to denote this gauge field, in [25] and many other publications that letter denoted the covariant derivative, which we will mark by careted (capped) derivation mark $\hat{\partial}$.

hand, and that our operators should effect complex $\psi(x_\nu)$ fields which depend on the four space-time co-ordinates, on the other.

We extend the role of the co-ordinates to a set of generalised variables alike Noether [32] did. These variables may be the four space-time co-ordinates or they may be others (and their number may vary). In her mathematical terms of invariant variational problems, the space-time co-ordinates did not play a distinguished role. According to her second theorem, other variables, among others (e.g., velocity-space co-ordinates), are allowed which may implicitly depend on the space-time co-ordinates. For practical reasons we replace the $f(\dot{x}_\mu, x_\nu)$ dependence with a $f(\dot{x}_\mu(x_\nu))$ dependence. The localisation is present here too (in the above generalised, Noetherian sense), although it makes us possible another way of calculating it.

We were seeking for invariance between scalar fields and (gauge) vector fields that describe kinetic processes, the latter depending therefore primarily on velocity. For this reason, we consider Lagrangians which depend on matter fields φ_k, and gauge fields $D_{\dot{\mu},\alpha}$, which all depend – in simple mathematical terms – on parameters. In physical terms these parameters are generally identified with the four space-time co-ordinates. In our specific case the dependence of **D** on x_μ will be given by the formula: $D_\mu = D_\mu(\frac{\partial x^\mu}{\partial x_4})$. The 2nd theorem of Noether is just about Lagrangians, which depend on arbitrary number of fields with arbitrary finite number of derivatives by arbitrary number of parameters. We can apply her theorem here because in mathematical terms she did not specify either the physical-mathematical character or the number of applicable parameters. Our consideration will be justified by the final result, which demonstrates that in a boundary situation, namely in the absence of a velocity-dependent gauge field we obtain the same currents that were derived in a space-time dependent field, (cf. Eqs. (4) and (7) below). In other words, in the absence of relativistically high velocities or acceleration, the effect of the velocity dependent gauge field can be neglected, and we get back to the same currents as derived in the semi-classical, only space-time dependent gauge's case. At the same time, in the presence of a velocity dependent gauge field, we derived new conserved Noether currents [18].

Noether's currents for gauge invariance localised in the velocity space

The presentation discusses general, non-Abelian case. Let's first introduce a (kinetic) **D** field localised in the velocity space. We introduce a λ_μ^ν tensor, which characterizes the changes of the velocity-space components in the space-time. Localization will be taken into consideration in this way (we refer to the generalized interpretation of localization as defined above).[4]

In general, we base on a transformation group *G* and the transformations of its elements, where the number of parameters are arbitrary finite numbers $(\alpha = 1,\ldots,\rho)$; , $(\beta = 1,\ldots,\sigma)$. The *p* are parameters on which the transformations, constituting the

[4] Relativistic covariance under Lorentz transformation $S(\Lambda)$ and its consequences are a standard part of quantum field theory textbooks for long, e.g., [33, Sec. 2.1.3]. Here we take into account time derivatives of Lorentz transformed velocities.

group elements, depend. They take the form of functions $p_{\alpha}(x_{\beta})$ and their derivatives. The group transformations depend on *p* and are finitely differentiable. *G* may take the form of different groups, depending on the concrete form of interaction in subject, namely *SO*(3,1), *U*(1), *SU*(2), *SU*(3) in the cases of the fundamental physical interactions.

We consider a Lagrangian density $L(\varphi_k, D_{\dot{\mu},\alpha})$, where φ_k, (k = 1, ..., n) are the matter fields - which also includes the velocity field $\dot{x}^{\mu} = \dot{x}^{\mu}(x_{\nu})$ - and $D_{\dot{\mu},\alpha}$,

($\alpha = 1,...,N$), are the (kinetic) gauge fields. We assume, that $L(\varphi_k, D_{\dot{\mu},\alpha})$ is invariant under the local transformations of a compact, simple Lie group *G* generated by T_{α}, ($\alpha = 1,...,N$) and $C^{\gamma}_{\alpha\beta}$ are the so-called structure constants, corresponding to the actually considered individual physical interaction's symmetry group.[5] For examples, in the case of *SU*(2) symmetry, *G* consists of 2×2 matrices with 3 independent components, representing a state doublet, and in the case of *SU*(3) its matrix has 8 independent components, representing a state triplet. For simplicity we assume that the matter fields belong to a single, *n*-dimensional representation of *G*.

The infinitesimal transformations of the matter- and the gauge fields determine the covariant derivatives of ψ in the gauge field. (For invariance, we can require that the derivatives of ψ coincide with the derivatives of $V\psi'$). The infinitesimal transformations can be formulated as follows:

$$\pi_3(S^2) = Z \qquad (k = 1, ..., n), \tag{1}$$

where the T_{α} are matrix-representation operators generating the group *G* and

$$\begin{aligned} s = \overline{\Psi}\Psi, \quad p = i\overline{\Psi}\gamma_5\Psi, \quad \vec{v} = \overline{\Psi}\vec{\lambda}\Psi, \\ \vec{a} = i\overline{\Psi}\gamma_5\vec{\lambda}\Psi, \quad j_{\mu} = \overline{\Psi}\gamma_{\mu}\Psi, \quad \tilde{j}_{\mu} = \overline{\Psi}\gamma_{\mu}\gamma_5\Psi, \end{aligned} \qquad (\alpha = 1,...,N) \tag{2}$$

where $\partial^{\dot{\rho}} = \frac{\partial}{\partial \dot{x}^{\rho}}$, and ג (Hebrew *g*, gimel) denotes a general coupling constant, which can be replaced by a concrete coupling constant for each individual physical interaction.

For the induced infinitesimal transformation δL of the Lagrangian density using the field equations for both the matter and the gauge fields, one obtains

$$\delta L = \partial_{\mu}\left(\frac{\partial L}{\partial(\partial_{\mu}\varphi_k)}\delta\varphi_k + \frac{\partial L}{\partial(\partial_{\mu}D_{\nu,\alpha})}\delta D_{\nu,\alpha}\right. . \tag{3}$$

One would like to describe the events, resulted in the interaction between the matter field and the kinetic (velocity-space dependent) gauge field, as they are observed from the usual 4D space-time. Therefore one needs to apply derivatives by the space-time co-ordinates.

We have derived from here the following two sets of equations:

$$J^{(1)\nu}_{\alpha}(x) = \partial_{\mu}F^{(1)\nu}_{\alpha} \qquad \partial_{\nu}J^{(1)\nu}_{\alpha} = 0 \tag{4}$$

$$J^{(2)\nu}_{\alpha}(x) = \partial_{\mu}F^{(2)\nu}_{\alpha} \qquad \partial_{\nu}J^{(2)\nu}_{\alpha} = 0 \tag{5}$$

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[5] We partly follow the clues by Higgs [30] and Weinberg [34] at the beginning of their papers with the exception that we consider different dependencies in the potential and kinetic Hamiltonian terms.

completed with

$$\frac{\partial L}{\partial(\partial_\mu D_{\dot{\nu},\alpha})}\lambda_\nu^\rho + \frac{\partial L}{\partial(\partial_\nu D_{\dot{\mu},\alpha})}\lambda_\mu^\rho = 0 \tag{6}$$

this set (4)-(6) demonstrates, that in the presence of a kinetic (velocity-dependent) gauge field, there exist two (families of) conserved Noether currents.

Although the two conserved currents are not independent, in the presence of a kinetic gauge field they exist simultaneously. (One can easily see, that λ_μ^ν mixes the components of the gauge-field currents depending on the 4D velocity space in a similar way, like the Lorentz transformation mixes the co-ordinates of four-vectors in the 4D space-time; since the λ_μ^ν tensor was defined to characterise the changes of the velocity-space components – accelerations – in the space-time.)

Taking into account the conditions how we have obtained these currents, one can write $J_\alpha^{(1)\mu}$ as

$$J_\alpha^{(1)\nu}(\dot{x}) = i\frac{\partial L}{\partial(\partial_\nu \varphi_k)}(T_\alpha)_{kl}\varphi_{kl} \tag{7}$$

The most significant conclusion of the above cited derivation (cf., [18]) is that in the presence of a kinetic gauge field **D**, there appear extra $J_\alpha^{(2)\nu}$ conserved currents. Taking into account conditions of the derivation of $J_\alpha^{(2)\nu}$, one can write it in the form

$$J_\alpha^{(1)\nu}(x) = i[\frac{\partial L}{\partial(\partial_\nu \varphi_k)}(T_\alpha)_{kl}\varphi_{kl}]\lambda_\nu^\mu - C_{\omega\beta}^\gamma D_{\omega\beta}(\dot{x})\lambda_\mu^\omega \times F_\gamma^{(2)\mu\nu}(x) \tag{8}$$

Their dependence on the velocity-space gauge is apparent, although, none of the conserved vector currents involve the gauge parameters $p_\alpha(\dot{x})$ and their derivatives.

From (4) and (7), considering consequences of (6), one obtains

$$\partial_\mu F_\alpha^{(1)\nu}(\dot{x}) = i\frac{\partial L}{\partial(\partial_\nu \varphi_k)}(T_\alpha)_{kl}\varphi_{kl}(\dot{x}) \tag{9}$$

From (5) and (8), considering the concrete forms of the covariant derivatives, one obtains

$$\partial_\mu F_\alpha^{(2)\nu}(\dot{x}) = i\frac{\partial L}{\partial(\partial_\nu \varphi_k)}(T_\alpha)_{kl}\varphi_{kl}(\dot{x})\lambda_\mu^\nu \tag{10}$$

Mathematical and physical conclusions

First conclusion – of the conserved Noether current (4) – is a conserved quantity: Conservation of the field charge (ד).

Second conclusion – of the conserved Noether current (5) – is another conserved quantity: Conservation of the isotopic field charge spin (Δ). Further, we could derive, in the usual way, the total isotopic field charge spin

$$\Delta = \frac{i}{\ }\int J_{ג}^{(2)4}\,\mathrm{d}^3x$$

which is independent of time and independent of Lorentz transformation. $J^{(2)\mu}$ does not transform as a vector, while Δ transforms as a vector under rotations in this isotopic field charge spin field.

Coupling of the two conserved quantities (ꓶ and Δ), what is based on the dependence of the two currents $J_{\alpha}^{(1)\mu}$ and $J_{\alpha}^{(2)\mu}$ on each other, has physical consequences. The quantities, whose conservation they represent, and which are coupled (by $\lambda_{\mu}^{\nu} = \partial_{\mu}\dot{x}^{\nu}$), exist simultaneously. *The derived conservation law verifies* just *the invariance between two isotopic states of the field charges, namely between the potential* $ꓶ_V$ *and the kinetic* $ꓶ_T$ what we intended to prove. We obtained, that *in the presence of kinetic fields we have two conserved currents that are effective simultaneously*. The kinetic gauge field **D** is present simultaneously with the interacting matter [φ] and gauge [**B**] fields. The presence of **D** corresponds to the property of the field charges ꓶ of the individual fields that they split in two isotopic states, and analogously to the isotopic spin, we named these two states *isotopic field charge spin* what we denoted by Δ. The source of the isotopic field charge spin (Δ) is the field $\varphi(\dot{x})$, in interaction with the kinetic gauge field **D**.

The physical meaning of Δ can be understood, when we specify the transformation group associated with the **D** field, which describes the transformations of ꓶ (i.e., the isotopic field charges). ꓶ can take two (potential and kinetic) isotopic states $ꓶ_V$ and $ꓶ_T$ in a simple unitary abstract space. Their symmetry group is $SU(2)$, that can be represented by 2x2 T_α matrices. There are three independent T_α that may transform into each other, where the structure constants can take the values 0, ±1. Let T_1 and T_2 be those which do not commute with T_3; they generate transformations that mix the different values of T_3, while this "third" component's eigenvalues represent the members of a Δ doublet. For the isotopic field charges compose a ꓶ doublet of $ꓶ_V$ and $ꓶ_T$, the field's wave function can be written as

$$\psi = \begin{pmatrix} \psi_T \\ \psi_V \end{pmatrix}. \tag{11}$$

(11) is the wave function for a single particle which may be in the "potential state", with amplitude ψ_V, or in the "kinetic state", with amplitude ψ_T. ψ in (11) represents a mixture of the potential and kinetic states of the ꓶ, and there are T_α that govern the mixing of the components ψ_V and ψ_T in the transformation. T_α (α = 1, 2, 3) are representations of operators which can be taken as the three components of the isotopic field charge spin, Δ_1, Δ_2, Δ_3 that follow the same (non-Abelian) commutation rules as do the T_α matrices, $[\Delta_1, \Delta_2] = i\Delta_3$, etc. These operators represent the charges of the isotopic field charge spin space, and ψ are the fields on which the operators of the gauge fields act.

The quanta of the **D** field should carry isotopic field charge spin Δ. The Δ doublet, as a conserved quantity, is related to the two isotopic states of field charges (ꓶ), and the associated operators (Δ_i) induce transitions from one member of the doublet to the other.

HOW TO INTERPRET THE ISOTOPIC FIELD CHARGE SPIN CONSERVATION?

Invariance between $\urcorner_V$ and $\urcorner_T$ means that they can substitute for each other arbitrarily in the interaction between field charges of any given fundamental physical interaction. They appear at a probability between [0, 1] in a mixture of states in the wave function ψ (11) so that the Hamiltonian of a *single particle* oscillates between V and T, while the Hamiltonian of a *composite system* is a mixture of the oscillating components of the particles that constitute the system. The individual particles in a *two-particle system* are either in the V or in the T state respectively, and switch between the two roles permanently; while the observable value of H is the expected value of the mixture of the actual states of the two, always opposite state particles.

Mediating bosons (*δ)*

The invariance between $\urcorner_V$ and $\urcorner_T$ (what is ensured by the conservation of $\varDelta$), and their abilitiy to swap, means also that they can restore the symmetry in the physical equations which was lost when we replaced the general $\urcorner$ (in our case mass m) by their isotopes $\urcorner_V$ and $\urcorner_T$ (concretely m_V and m_T).[6]

We denote the *predicted* quanta of the **D** field by δ. We call this hypothetical boson "*dion*", after the Greek term meaning ‘flee’, ‘flight’, ‘rout’ in English. The δ quanta (dions) carry the $\varDelta$ (isotopic field charge spin as a physical property: charge of the **D** field). According to the IFCS model, gravitational interaction takes place between two massive particles with the simultaneous exchange of a graviton and a dion.

Starting from the equivalence principle, through the qualitative distinction of the masses of gravity and inertia as isotopic field charges of the gravitational field and interaction between them, we concluded the prediction of a boson that mediates their interaction.

One of the main consequences of the isotopic field-charge spin conservation was the prediction of the dions, as bosons that mediate between the two isotopic states of the field charges. At the same time, experimental observation of dions my serve as a decisive test for the appropriateness of the presented theory.

[6] Consequences of the application of effective field theories were analysed e.g., in philosophy by E. Castellani [35] and in physics by S. Weinberg [23].

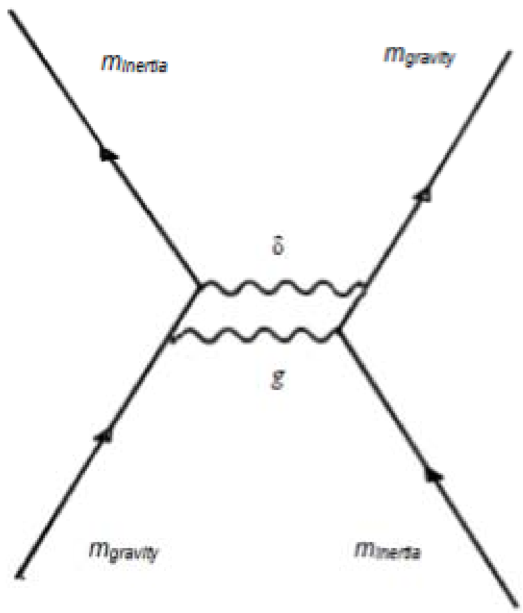
The presence of isotopic field-charges demands Finsler geometry

Another consequence is the appearance of Finsler spaces in the gravitational theory. Specify (9) for the gravitational field [36]! The right side of the equation contains the scalar field that serves for the source of the gravitational field. The ℷ can be replaced by the gravitational coupling constant *g*. As we noticed, the dependence on the gauge fields is on the left side of the equation (9). $F_{\alpha}^{(1)\mu\nu}(\dot{x})$ must satisfy the

$$T_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} + \frac{1}{4}\delta_{\mu\nu}g^{\kappa\sigma}F_{\lambda\sigma}g^{\lambda\rho}F_{\kappa\rho}$$

identity for the energy-momentum tensor $T_{\mu\nu}$. (In order to bring this form in compliance with the indices in (9), one should raise the indices by multiplying with the metric tensor $g_{\beta\gamma}$ in the right side.) This energy-momentum tensor $T_{\mu\nu}$ can be expressed by the way of the Einstein equation

$$T_{\mu\nu} = -\frac{1}{8\pi G_N}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}) \tag{12}$$



where $R_{\mu\nu}$ is the Ricci tensor defined by the help of the derivatives of the metric tensor $g_{\mu\nu}$, R is the Ricci scalar formed from the Ricci tensor (Riemann curvature) and the metric tensor, and Λ is a constant of Nature, as well as G_N the constant of Newton.

The metric tensor $g_{\mu\nu}$ and its derivatives depend on the localisation of the given point in the space-time in the General Theory of Relativity (GTR), and are subject of Riemann geometry. In the presence of a kinetic field, that means, isotopic mass field **D** (mass being the field-charge of the gravitational field), however, the curvature depends also on velocity. (Whose velocity? On the actual inertial velocity of a test unit-mass placed in a given space-time point in the reference frame fixed to the source of a scalar gravitational field φ which appears on the right side of (9).) The $g_{\mu\nu}$ metric tensor, and consequently the affine connection field and the curvature tensor formed from its derivatives, depend on space-time and velocity co-ordinates. With the appearance of the dependence on the velocity vector, the curvature becomes dependent on its direction in each space-time point. The direction (additional parameter) attributed to each space-time point is defined by the orientation of the velocity of a test unit-mass in the given space-time point, $\frac{\mathbf{v}}{|v|}$. The curvature can no more follow a “simple” Riemann geometry, it follows a Finsler geometry whose metric is defined by the dependence of $g_{\mu\nu}$ on (x_σ and) $\dot{x}_\rho$.

Of course, the space-time plus four-velocity dependence of the metric tensor $g_{\mu\nu}$ affects its all derivatives, including the formation of the affine connection field (from first derivatives) and the Riemann curvature (or Ricci tensor, second, covariant derivative)

$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}\left[\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu}\right] \qquad \Gamma^{\lambda}_{\mu\nu} = g^{\lambda\rho}\Gamma_{\rho\mu\nu}$$

and

$$R_{\mu\nu} = \partial_\mu \Gamma^{\lambda}_{\nu\lambda} - \partial_\lambda \Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda} - \Gamma^{\lambda}_{\sigma\lambda}\Gamma^{\sigma}_{\mu\nu}$$

The solution of the Einstein equation in velocity dependent field with Finsler geometry must necessarily lead to solutions different from those based on the "classical GTR" in permanent curvature space (like the solution by Schwarzschild and its later corrections).

WHAT ROLE DOES THE CONSERVATION OF THE ISOTOPIC FIELD-CHARGE SPIN PLAY?

The role of equation (12) is to retain the invariance between the two isotopic forms, namely gravitational and inertial, of masses. The importance of this is to save the covariance of our equations. Since there appear two different kinds of (isotopic) masses in the energy-momentum "four-vector" (in the fourth column of $T_{\mu\nu}$,) it does no more transform as a vector, and Lorentz transformation can no more guarantee alone the covariance of our equations.

As a consequence of the distinction between m_V and m_T, as well as the association of the energy content with the mass m_V and the components of the momentum with m_T, we lose also the symmetry of the $T_{\mu\nu}$ energy-momentum tensor. To retain symmetry in Einstein's field equations we must require again the invariant transformation of m_V and m_T into each other in an appropriate gauge field, namely in **D**. We refer to [25] who foresaw the possible generalization of YM type gauge invariance in general relativity "in close analogy with the curvature tensor". If we consider the energy-momentum tensor (in which both isotopic states of mass appear) as the source of the gravitational field, then – in the usual way – the scalar and the vector potential can be separated. See, m_4 in T_{44} does not compose a fourth component of a four-vector in the classical theory of gravitation where there is a single scalar mass. If we consider now $m_4 = m_V$, the three components of the kinetic mass m_T can compose a three-vector, however $T_{\mu 4}$ will not form a four vector either.

To maintain the Lorentz invariance of our physical equations in the gravitational field, we must demand to restore the invariance of $\begin{pmatrix} \vec{m}_T \\ m_V \end{pmatrix}$ under an *additional transformation* that should *counteract the loss of symmetry caused by the introduction of two isotopic states of mass*. We discussed that transformation in section 2. Further, in the case of gravitation the relation of the scalar and the vector fields are not linear even if we have not made distinction between the potential and kinetic masses. The non-linearity is coded in the relation of the tensors [26] at the right side of the Einstein equation (12) (in units c = 1), or we can write $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ where the Einstein tensor is defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ whose covariant derivative must vanish.

Since our $T_{\mu\nu}$ tensor has already lost its symmetry, we can take $\Lambda g_{\mu\nu}$ into account within a modified $T'_{\mu\nu}$ – handling the gravitational and kinetic masses in it together with the dark energy – and we get the following formally symmetric equation: $G_{\mu\nu} = 8\pi G T'_{\mu\nu}$.

The symmetry of the energy-momentum tensor can be saved by the invariant gauge transformation of the IFCS. The most important analogy is between the behaviour of the potential and the kinetic field charges of the individual fields that makes probable to postulate a unique transformation to assure their invariance (cf., section 2).[7] So the invariance under the Lorentz transformation combined with the invariance of the isotopic field charge spin field provide together the covariance of the gravitational equation. However, this combined transformation should now be taken into consideration in a field with a metric depending on all space-time and velocity co-ordinates, following a Finsler geometry.

Comparison of the invariance properties in classical GTR and in the IFCS model

In classical physics, conservation laws – as consequences of the invariance properties of the investigated systems – can be obtained by integration of the Euler-Lagrange equations of motion of classical mechanical point systems. According to Hamilton's principle the variation of the action integral of the system's Lagrangian must be zero. These conservation laws include the conservation of the energy – invariance under translation in time. That conserved energy is equivalent with a well determined amount of mass $E = mc^2$, where $m = m_V$ is gravitational mass, and this conservation law does not provide any information on the quantity of kinetic mass.

In general relativistic treatment, the source of the gravitational field is the $T_{\mu\nu}$ momentum-energy stress tensor, which includes the sources of inertial and gravitational effects as well. Applying the same variational method and integration for the Einstein equation (using [+ + + –] signature) we derive the conservation of the $-T_{44}$ element of the $T_{\mu\nu}$ momentum-energy stress tensor. $-T_{44}$ is energy density of the gravitational field, and is proportional to a certain amount of mass.

[7] As [27] stated, "In contrast to the symmetry or *invariance* requirement in STR, the principle in GTR is most often presented as strictly speaking a *covariance* requirement." Gauge theories behave like GTR, at least in this respect. General covariance "is not tied to any geometrical regularity of the underlying spacetime, but rather the form invariance (covariance) of laws under arbitrary smooth coordinate transformations" [27, p. 34]. [28] found that the more general geometry resulting from admitting local changes called gauges described not only gravity but also electromagnetism. He showed also that the conservation laws of Noether follow in two distinct ways in theories with local symmetries. This led to the Bianchi identities, which hold between the coupled equations of motion, and which are due to the local gauge invariance of action. Later [29] demonstrated that the conservation of the electric charge followed from the local gauge invariance in the same way as does energy-momentum conservation from co-ordinate invariance in GTR.

According to invariance under translations in the Minkowski space (Lorentz transformation) the conserved current can be written in the form

$$\partial_\mu T_{\mu\nu} \equiv \partial_\mu \left(L\delta_{\mu\nu} - \partial_\nu \varphi_r \frac{\partial L}{\partial \partial_\mu \varphi_r} \right) = 0$$

where φ_r denote functions on which (and their first derivatives) the Lagrangian may depend.

The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

provides the elements of Tμν in which – according to the left side – the contribution of the kinetic and potential components are mixed by the gμν curvature tensor. Applying the usual integration method and Gauss' theorem, we get the fourth column of the momentum-energy stress tensor for a conserved quantity, what is no else than the four-momentum density, which behaves like a four-vector and whose individual components are

$$P_\nu = \frac{1}{ic} \int T_{4\nu} \, \mathrm{d}V$$

or separated

$$P_k = \frac{1}{ic} \int T_{4k} \, \mathrm{d}V = \frac{1}{ic} \int \partial_k \varphi_i \frac{\partial L}{\partial \partial_4 \varphi_i} \mathrm{d}V \qquad (k = 1, 2, 3);$$

$$H = -icP_4 = -\int T_{44} \, \mathrm{d}V = \int (\partial_4 \varphi_i \frac{\partial L}{\partial \partial_4 \varphi_i} - L) \, \mathrm{d}V$$

what are considered the conserved total momentum and energy of the field respectively.

If we take into account the qualitative difference between the masses mT (what appear in the components of Pk) and mV (what appears in H) that are mixed by the curvature tensor gμν in the elements of Tμν, this consideration will involve the mixed mT and mV dependence of the Lagrangians as well. As a consequence, Pk and H cannot be considered separately, and independently of each other, conserved quantities. (We do not investigate here the ambiguous interpretations of invariant mass.) The covariance of the gravitational equation can no more be secured by the Lorentz invariance alone. The lost symmetry of nature can be restored only with the shown invariance between the isotopic mass states (as field charges of the gravitational field, conservation of Δ) which are rotated in an isotopic field charge spin gauge field. The covariance of the gravitational equation is a result of invariance under the combination of the Lorentz transformation and rotation in the isotopic field charge field. In the latter case the four components of (Pk[mT], H[mV]) transform as isovectors. Due to the IFCS gauge transformation, the transformation of the field components can be described in a (space-time +) velocity dependent gauge field, whose metric, consequently, depends also on the velocity components, and is subject of a Finsler geometry.

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A cluster calculation for ^{6}He spectrum

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Abstract. The ^{6}He nucleus is considered as the cluster $\alpha+n+n$ system. The excitation energies of the low-lying levels for this nucleus are calculated using the configuration-space Faddeev equations. The analytical continuation method in a coupling constant is applied for calculation of resonance parameters. The αn interaction is constructed to reproduce the results of R-matrix analysis for αn-scattering data. A realistic AV14 potential describes the nn interaction. Additional three-body potential adjusted by the ground state energy of ^{6}He is used. The energies of the low-lying resonances of ^{6}He (0^+, 2^+, 1^-, 2^-) are reasonably reproduced by the calculations.

Keywords: Properties of nuclei; nuclear energy levels; 6 ≤ A ≤ 19; Cluster models; Few-body systems.
PACS: 21.10.-k; 27.20.+n; 21.60.Gx; 21.45.-v.

INTRODUCTION

We study the cluster phenomena in light nuclei [1] using three-body cluster models. Particularly, the cluster $\alpha+n+n$ and $\alpha+\alpha+n$ systems are considered as appropriate for description of the ^{6}He and ^{9}Be nuclei. In the present work we focus on a model for the αn inter-cluster interaction, which has to reproduce the set of the experimental data as well as the αn scattering data and low-lying spectrum of ^{6}He and ^{9}Be. In Ref. [2] we proposed an αn potential that was constructed to reproduce the results of R-matrix analysis for αn-scattering data [3]. This potential is a modification of the potential given in [4]. The potential allows us [2] to describe the low-lying spectrum of ^{9}Be closely to the experimental data. The same αn-potential is applied for description of low-lying spectrum of the ^{6}He nucleus. We show that the energies of the low-lying resonances of ^{6}He (0^+, 2^+, 1^-, 2^-) are reasonably reproduced by our calculations.

Spectral properties of this nucleus have been calculated within cluster model in a number of works [4-10]. It has to be noted that the present calculations reproduce the energy of the 1^- levels evidenced by the experiments [11]. These levels have not been found by the previous calculations [8-10], except by the calculations [4] that, however, are far from the experimental data.

Short description of the model assumptions and the methods of calculations: The calculations are based on the Faddeev equations in configuration space. The LS scheme is used for partial wave analysis and the model space is restricted to the states with the total spin $S=0$, due to a weak interaction between nucleons in the spin-isospin state "triplet-triplet" ($s=1$, $t=1$) that corresponds to the total spin of the system $S = 1$. To evaluate the parameters of the resonances, applied is the analytical continuation method in a coupling constant. The constant is the strength parameter of non-physical three-body potential [2]. Additional adjustment of the model was made to reproduce the experimental value for the ground state of the ^{6}He nucleus. As it is well known previously [4-8] the three body model calculations with pair potentials do not lead to the experimental value. Following [4-8] we adjust the calculated ground state energy of the $\alpha + n + n$ system using three-body potential given as one range Gaussian with attraction. This effective potential forms essentially the low-lying spectrum of the $\alpha + n + n$ system.

MODEL AND METHODS

The ^{6}He nucleus is described by three body $\alpha + n + n$ model as two-neutron halo nucleus. Strong clusterization of the alpha-particle is assumed in the nucleus. The bound state of the nucleus is the 0^+ state. Due to the Pauli principle for the system of five nucleons, the s-wave configuration of the αn subsystem is suppressed for the system, since the p-shell configuration dominates. The $\alpha + n$ and $n + n$ subsystems of $\alpha + n + n$ are unbound: it is the Borromean type of nucleus [4].

Our calculations are based on the Faddeev equations in configuration space. The total wave function of the $\alpha + n + n$ system is decomposed to the sum of the Faddeev components U and W that correspond to the two channels $(nn)\alpha$ and $(\alpha n)n$ of the particles rearrangement, respectively:

$$\Psi = U + (I + P)W ,$$

P is permutation operator for identical particles. In this notation the differential Faddeev equations are given as

$$\begin{aligned} (H_0 + V_{nn} - E)U &= V_{nn}(W + PW), \\ (H_0 + V_{\alpha n} - E)W &= V_{\alpha n}(U + PW). \end{aligned} \qquad (1)$$

The LS scheme is used for partial wave analysis of the equations [2,12]. The LS basis allows us to restrict the model space to the states with the total spin $S = 0$. The possible configurations with $S = 1$ are not taken into account in our calculations. It can be noted that αn potential used for this calculation has spin-orbital component that fixes the configuration $S = 0$ and $S = 1$ within the total momentum representation. According to the evaluations of different authors the total contribution of the $S = 1$ configuration is ranged from 5% to 14% for the $\alpha + n + n$ ground state [5].

The αn interaction $V_{\alpha n}$ is taken into account in s, p and d states. The p and d-wave components include central and spin-orbit parts: $V_{\alpha n}{}^l(r) = V_c{}^l(r) + (s,l)V_{so}(r)$. The coordinate dependencies of the components have the form of one- or two-range Gaussians [2]. The s-wave component is repulsive [3,6,13] to simulate the Pauli exception for αn in the s-state. This αn-potential proposed in our work [2] is modified potential given in Ref. [4].

The Argonne V14 (AV14) potential [14] was used for the nn-interaction. Comparing the potentials of singlet-triplet ($s=0$, $t=1$) and triplet-triplet ($s=1$, $t=1$) spin-isospin states it was assumed that the triplet ($s=1$) components are essentially weaker. According to this fact we ignored the $S=1$ three-body wave function component.

An effective three-body force simulating the effects of violation of the strong cluster structure of ^{6}He [4,6] is taken into account. Two reasons for such violation may be noted: the first is that the ^{3}H+^{3}H threshold is close to the energy region considered (~12 MeV) [5] and the second is the use of the cluster model with "frozen core" [15]. The three-body L-independent potential is defined by one range Gaussian: $V_{3bf}(\rho) = V_0 \exp(-\alpha\rho^2)$. Here ρ is hyper-radius of the tree-body system: $\rho^2 = x^2 + y^2$, where x, y are the mass scaled Jacobi coordinates [2]. Adding this potential into left hand side of the Faddeev equations (1) one can reproduce the experimental value of the ^{6}He binding energy. The adjustment is governed by the two free parameters V_0 and α.

Bound state problem formulated by the configuration space Faddeev equations (1) was numerically solved applying the finite deference method with the spline collocations [16]. The eigenvalues are calculated by the method of inverse iterations was used.

The method of analytical continuation in a coupling constant was used to calculate parameters of the resonances. The coupling constant is the depth of non-physical tree-body potential having a form of one range Gaussian. This potential is considered as perturbation corresponding Hamiltonian and is added to the left hand of the equations (1). The potential has form: $V_3(\rho) = -\delta \exp(-\alpha\rho^2)$ with parameters α, $\delta \geq 0$ that can be varied. For each resonance there exists a region $|\delta| \geq |\delta_0|$ where a resonance becomes a bound state. In this region we calculated $2N$ bound state energies corresponding to $2N$ values of δ. The continuation of this energy set as a function of δ onto complex plane was carried out by means of the Pade approximant [2]: $-\sqrt{E} = \sum_{i=1}^{N} p_i \zeta^i / (1 + \sum_{i=1}^{n} q_i \zeta^i)$, where $\zeta = \sqrt{\delta_0 - \delta}$. The complex value of the Pade approximant for δ=0 gives the energy and width of resonance: $E(\delta = 0) = E_r + i\frac{\Gamma}{2}$. Accuracy of the Pade approximation for the resonance energy and width, depends on the distance from the three body threshold, and orders N of the Pade approximants.

RESULTS

Using the three-body s-wave potential defined by the parameters: $V_0 = -1.661$ MeV and $\alpha = 0.2$ fm^{-2} we obtained for the $\alpha + n + n$ ground state -0.9725 MeV, which is close to the binding energy of ^{6}He (-0.973 MeV) [11]. The orbital momentum configurations taken into account for this calculation are shown in Table 1. The calculation convergence for the binding energies relative to increasing model space is fast. The main contribution that provides the bound state of the system is coming from the p - wave of the αn interaction, due to attractive p - wave component of the αn potential [2].

Note that the calculation without the three-body potential does not produce bound system.

TABLE 1. Orbital momentum configurations for two type of the Faddeev components ($(nn)\alpha$ or $(\alpha n)n$) for the 0^+ state (total spin $S=0$) and binding energy E of $\alpha+n+n$ system. l is orbital momentum pair of particles, λ is orbital momentum, relative motion of the third particle to the center of mass of the pair.

	$(nn)\alpha$	$(\alpha n)n$	**E (MeV)**
l	**0 2 4**	**0 1 2**	
λ	**0 2 4**	**0 1 2**	**-0.973**
l	**0 2**	**0 1 2**	
λ	**0 2**	**0 1 2**	**-0.972**
l	**0**	**0 1 2**	
λ	**0**	**0 1 2**	**-0.968**
l	**0**	**0 1**	
λ	**0**	**0 1**	**-0.953**
l	**0 2**	**0 1**	
λ	**0 2**	**0 1**	**-0.957**
l	**0 2 4**	**0 1**	
λ	**0 2 4**	**0 1**	**-0.957**
l	**0 2 4**	**0**	
λ	**0 2 4**	**0**	**not bound**

The calculations for the resonances are performed for the 2^+, 1^-, 2^- states. Table 2 presents the orbital momentum configurations for the Faddeev components corresponding to the $(nn)\alpha$ and $(\alpha n)n$ rearrangement channels, which are taken into account.

TABLE 2. Orbital momentum configurations for two type of the Faddeev components ($(nn)\alpha$ or $(\alpha n)n$) for the J^π state. l is particles pair orbital momentum, λ is orbital momentum of relative motion of the third particles toward the center of mass of the pair.

	$(nn)\alpha$	$(\alpha n)n$
2+		
l	**0 2 2 2 4**	**0 1 1 2 2 2**
λ	**2 0 2 4 2**	**2 1 3 0 2 4**
1-		
l	**0 2 2**	**0 1 1 2**
λ	**1 1 3**	**1 0 2 1**
2-		
l	**2 2**	**1 2 2**
λ	**1 3**	**2 1 3**

The results for the parameters of the resonances are presented in Table 3, where we give also comparison with the experimental data [11] and results of other authors [7-6]. The 2^+_1 resonance, being the first excided state of ^{6}He, has been well studied experimentally [11]. Others resonances are not reliably defined. In particular, the existing the 1^- resonance is discussed [8].

The calculated resonance widths are comparable to the experimental data; the narrow resonance (2^+_1) is reproduced as relatively narrow one, the broad resonances are calculated as broad ones. The best fit for measured energy we obtained for the 0^+ and 1^- levels. The same accuracy for the energy of the levels of 2^+_1 and 2^- was not be reached.

TABLE 3. Ground state (0^+_1) energy and parameters of the resonances (E_r, Γ) for the 0^+, 2^+, 1^- states in ^{6}He. Energy is measured from the $\alpha + n + n$ threshold.

State J^π	**Calculations** (E_r, Γ)	**Exp.** (E_r, Γ)
0^+_1	**-0.973**	**-0.973**
2^+_1	**(2.(1), 1)**	**(0.822 ± 0.025, 0.113 ± 0.020)**
1^-_1	**(4.3, 6)**	**(4.6 ± 0.3, 12.1 ± 1.1)**
0^+_2	**(4.5, 5)**	
2^+_2	**(4.9, 9)**	
1^-_2	**(13.(3), 12)**	**(13.6 ± 0.7, 7.4 ± 1.0)**
2^-_1	**(17, 16)**	

The present calculations predict two levels for each 0^+, $2^+, 1^-$ state. We compare our results for the energy spacing between the first and second level of these states with the experimental data and others calculations. The results are listed in Table 4. Again we see good agreement between our results and the experimental data. Note that the previous calculations do not reproduce the experimental results for the 1^- state.

TABLE 4. Results for the energy spacing between the first and second level of each state 0^+, 2^+ and 1^-.

	$(E_2 - E_1)(0^+)$, **MeV**	$(E_2 - E_1)(2^+)$, **MeV**	$(E_2 - E_1)(1^-)$, **MeV**
Danilin et al. [7]	**6.38**	**1.9**	**not found**
Kato [8]	**4.67**	**1.54**	**not found**
Pieper [17]*	**6.1**	**2.8**	**---**
Our	**5.5**	**2.8**	**9.0**
Exp. [11]	**5.6**	**3.8**	**9.0 ± 0.7**

*GFMC calculation with the AV18+IL2 potentials.

From the comparison we can generally conclude that new proposed potentials for αn-interaction and three-body force gives the reasonable description for the low-lying spectrum of ^{6}He. The illustration for this statement is presented in Fig. 1 where the experimental data for low-lying energy levels of ^{6}He from [11] and results of the calculations are shown together.

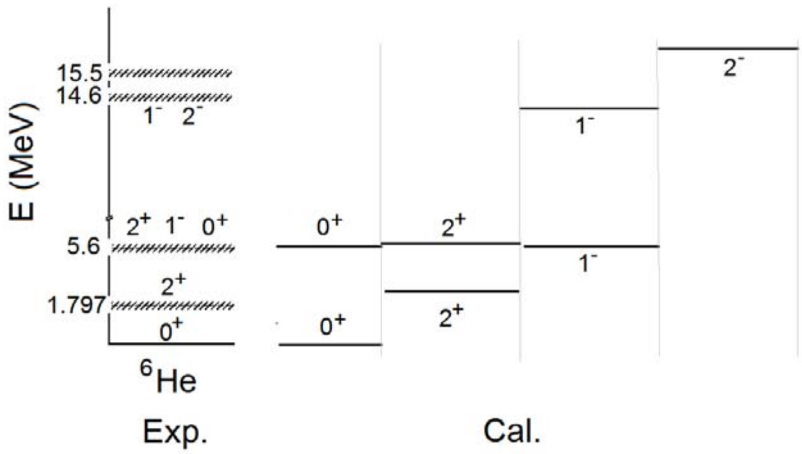

FIGURE 1. Experimental data and results of the calculations for low-lying energy levels of ^{6}He.

We used Pade approximation with the method of the analytical continuation in coupling constant. Details of the numerical procedure are illustrated in Fig. 2. The real part of the Pade approximants for the 0^+, $1^-, 2^+, 2^-$ states as a function of the δ parameters is shown. Calculated resonance energies correspond to the value $\delta = 0$.

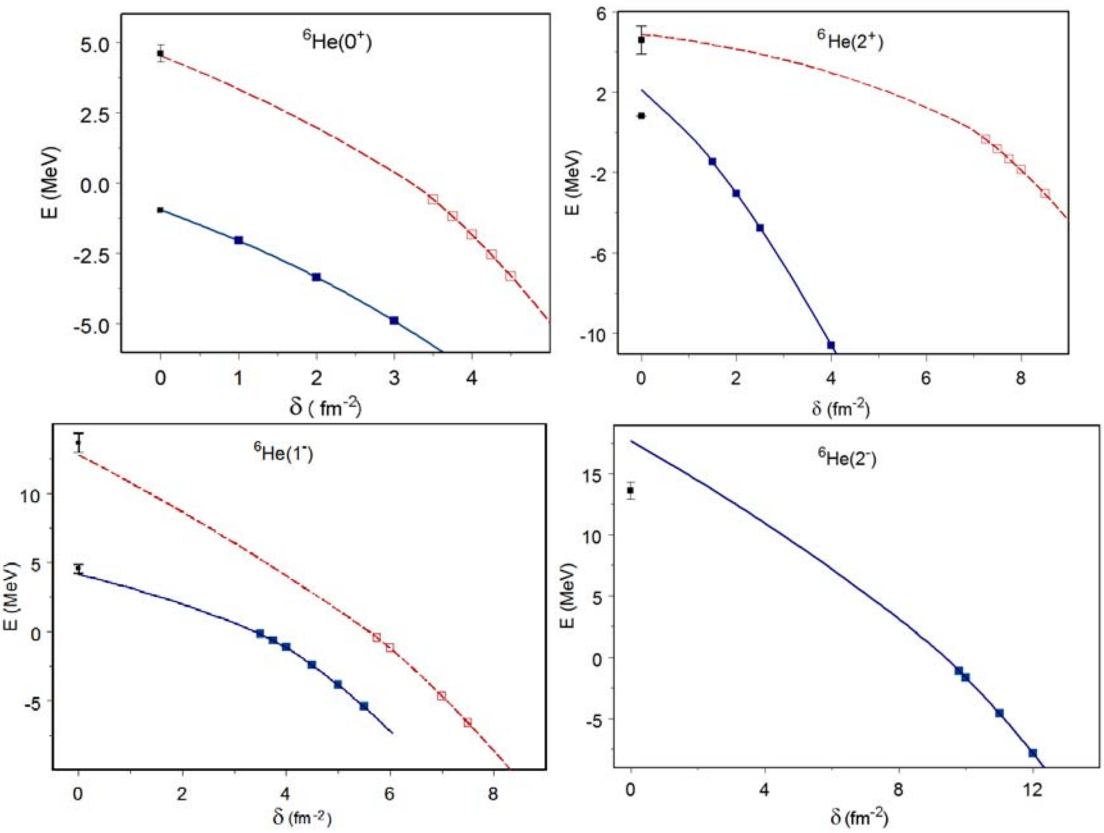

FIGURE 2. Real part of the Pade approximants for the 0^+, $1^-, 2^+, 2^-$ states (solid and dashed lines). Calculated resonance energies correspond to the value when $\delta = 0$. The energies of bound

states for $\delta > 0$ are noted by the open and filled squares. Experimental data are shown with experimental errors. Energy is measured from the $\alpha + n + n$ threshold.

It has to be noted that the method used for calculation of energy of the resonances may lead to some solutions which are not related to the initial problem, due to arbitrary three-body potential used. These "non-physical solutions" can be separated by applying different range parameter α for the three-body potential. The situation when the different values of the α parameter give the different resonance energy means that it is "non-physical solution". We made the test for the energy of 1^-_1 resonance. In Fig. 3 the result of the Pade-calculations for energy of the 1^-_1 resonance are presented for different α parameters. The same value was obtained for $\alpha = 0.2$ fm^{-2} and $\alpha = 0.25$ fm^{-2}. This fact confirms that our results for the 1^-_1 resonance energy are correct.

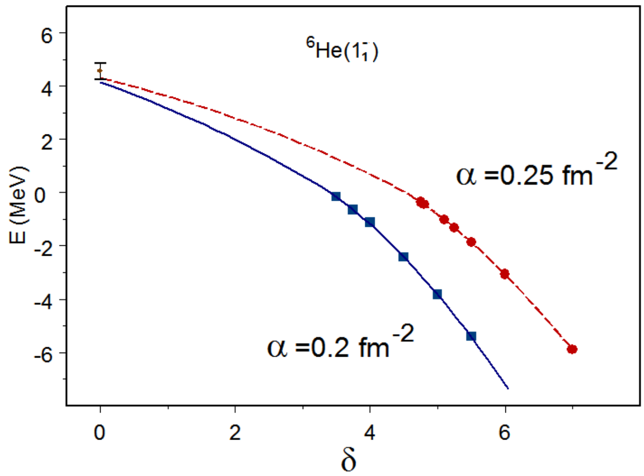

FIGURE 3. Real part of the Pade-approximants calculated for the 1^-_1 state. Different values of the α parameters of the non-physical three body potential are used: $V_3(\rho) = -\delta \exp(-\alpha\rho^2)$.

Our result for energy of the 2^+_1 level disagrees with the experimental data given with a good accuracy [11]. To explain this disagreement, we evaluated an influence of the three-body potential parameters on the 2^+_1 level resonance energy. We changed the parameters of the three-body potential as following $V_0 = -3.011$ fm^{-2}, $\alpha = 0.3$ fm^{-2}. This potential reproduces the ground state energy of ^{6}He. New calculated value for the energy of the 2^+_1 resonance is changed insignificantly from the previous result and is about 2 MeV. We found that the 2^+_1 energy, closed to the experimental data, can be obtained by the strength parameter change to the value of -4.4 fm^{-2}. The parameters of the 2^+_1 resonance with this change are E_r=0.9 MeV and Γ=0.1 MeV. Thus, an effective three-body potential may be modified so that the calculations reproduced the experimental data for the 2^+_1 level. Obviously this potential cannot reproduce energies of other levels. For example, the energy of the 0^+_1 ground state calculated with this potential is -2.707 MeV. We conclude that effective three-body interaction ought to have complex dependence on orbital momentum configuration of the $\alpha + n + n$ system. In particular, the 2^+_1 resonance is a narrow resonance that differs from other resonances considered. As we see above the range parameter of the effective three-body potential for this state must be larger to simulate this small resonance width. Thus one can assume that the spatial region of interaction of particles is more compact for the 2^+_1 state.

The disagreement between the calculated 2^- resonance energy and experimental data can be related with increasing numerical error of the analytical continuation in a coupling constant method for the large resonance energies (about E_r=12 MeV).

CONCLUSION

We have shown that reliable description of the ^{6}He low-lying spectrum within the cluster model is possible using the αn potential proposed in [2] together with the three-body potential applied in the presented work. We found that the singlet spin configuration ($S = 0$) is dominated for 0^+, 1^-, 2^+ and 2^- states. Disagreement between calculated energy of the 2^+_1 level and the experimental data may be related with choice of the effective three-body potential used for rough simulating of the complex three-body interaction having place in the ^{6}He nucleus considered as the cluster $\alpha + n + n$ system.

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Analytical Consideration of Quantum-Confined Stark-Effect and Intersubband Optical Transitions in Semiconductor Quantum Well

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Abstract. In the regime of strong quantization the single-electron states are considered theoretically in the wide-band semiconductor film placed in a homogeneous electrostatic field. For a certain range of values of the external field the explicit expressions were obtained for the energy spectrum and envelope wave functions of charge carriers in the film. The dependence of the parameters of direct intersubband electro-optical transitions in the film on the intensity of the external field was also considered.

Keywords: dipole transitions, electroabsorption, electric field, semiconductor film, quantum well.

PACS: 78.21.Fg, 78.67.De, 78.20.Jq

1. INTRODUCTION

The study of low-dimensional semiconductor structures has both purely scientific and practical meaning, and is dictated by the needs of rapidly progressing modern nanotechnology [1-3]. In quantum wells of thin films, particularly, the size quantization along one direction leads to the appearance of new properties of a two-dimensional subsystem of current carriers, fundamentally differing from the case of bulk sample; it explains their wide application in various quantum equipments as an active element [3-7]. As is known, the external electric field, along with the geometric characteristics of the system, is one of the most powerful modulating factors, enabling purposeful control of the energy spectrum of charge carriers and other characteristics of the band structure of sample [8-11]. Particularly, the modulation of the optical absorption in quantum wells by means of external electrostatic field lies in the fundament of the quantum electro-optical modulators and photo detectors [12-16]. Hence, it is clear that of certain interest is the theoretical study of the behavior of charge carriers in quantized films in the presence of an external electric field, - with a focus of finding the right solution of the problem in analytical form, which provides opportunities to identify and predict the conditions necessary for the effective formulation of an appropriate experiment. It's worth noting that on a purely theoretical level representation of the solution of the problem of electro-optical transitions in quantum well in an analytical form is possible only in limited cases of weak and strong fields, when the energy acquired by the particle from the field, respectively, is much smaller or much larger than the particle's energy of size-quantization (see, for example, Ref. [8] and references cited therein). In this theoretical work expressions are obtained in an analytical form for the energy spectrum and wave functions of charge carriers in the film at a time when the external - field, directed along quantization axis, varies from zero up to values at which the energy acquired by the particle from the field becomes comparable with its energy of size quantization in the film. For this range of fields the electro-optical intersubband transitions in the film are also considered.

2. SINGLE-PARTICLE STATES IN FILM IN THE PRESENCE OF EXTERNAL FIELD

The dimensions of the film along the axes x, y are supposing unlimited, and along the quantization axis (z) we approximate the film by infinitely deep potential well [8]. We assume the external homogeneous field with intensity $\vec{E}$ directed along the positive direction of the quantization axis of the film. Taking into account the sign of electron's charge $(q=-|e|)$ for its potential energy $U(z)$ in the film we will ha have:

$$U(z)=\begin{cases}\infty; & z \le 0,\, z \ge L \\ Fz; & 0 \le z \le L\end{cases} \tag{1}$$

where $F=qE_z$ is the force effected by the field on the charge, and the potential energy $U(z)$ is normalized by the initial condition $U(z=0)=0$. The thickness of the film L is assumed much smaller than the Bohr radius of *3D* exciton a_B:

$$L << a_B \tag{2}$$

Taking into account the condition (2), the Schrödinger equation for the particle motion along the z axis can be written as follows:

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dz^2}\psi(z)+Fz\psi(z)=E\psi(z); \quad \psi(z=0)=\psi(z=L)=0 \tag{3}$$

where μ is the particle effective mass, E is its energy along the quantization axis. From general considerations it is clear, that in this case, depending on the value of the external field, energy levels of the particle $E_n, (n=1,2,3,...)$ may have either higher or lower values of $E_0=F_0L=U(z=L)=U_{\max}(z)$. In this paper, we consider the case, when $E_n > FL$. Physically, this case corresponds to a range of values of the external field, where the energy imparted to the particle by the external field is less or of the order of the energy of the size-quantization of particle in the film. Introducing the notations

$$\frac{2\mu E}{\hbar^2} = -\lambda^2, \frac{2\mu F}{\hbar^2} = \beta, \beta z = g(z),$$

the equation (3) will have the following form

$$\frac{d^2\psi(z)}{dz^2} - \left[\lambda^2 + g(z)\right]\psi(z) = 0; \tag{4}$$

By substituting

$$\psi(z) = \exp\left[\lambda \int u(z)\, dz\right] \tag{5}$$

equation (4) becomes Riccati like equation [17]:

$$\lambda \frac{du(z)}{dz} + \lambda^2 u^2(z) - \lambda^2 - g(z) = 0 \tag{6}$$

The solution of this equation can be represented as a series

$$u(z) = \sum_{k=0}^{\infty} u_k(z)\lambda^{-k} \tag{7}$$

which must converge [17]. Substituting (7) in (6), for the first three values $u_k(z)$ we obtain:

$$u_0 = \pm 1;\ u_1 = \frac{du_0}{dz} = 0;\ u_2 = \frac{g(z) - u_1' - u_1^2}{2u_0} = \frac{\beta z}{2u_0}; \tag{8}$$

To calculate the rest of the series with $k \geq 2$ we will use the following recurrence relation [17]:

$$u_{k+1} = -\frac{1}{2u_0}\left[u_k' + \sum_{p=1}^{k} u_p u_{k+1-p}\right] \tag{9}$$

after which for the subintegral function (5) we obtain the following sequence:

$$\lambda u(z) = \lambda + \frac{\beta z}{2\lambda} - \frac{\beta}{4\lambda^2} - \frac{\beta^2 z^2}{8\lambda^3} + \frac{\beta^2 z}{4\lambda^4} + \left(\frac{\beta^3 z^3}{16} - \frac{5\beta^2}{32}\right)\frac{1}{\lambda^5} - \frac{\beta^3 z^2}{4\lambda^6} + \quad \ldots \tag{10}$$

Passing to the dimensionless energy parameters

$$\eta = \frac{FL}{\varepsilon_1}, \left(\beta = \frac{\eta\pi^2}{L^3}\right);\ \alpha = \frac{E}{\varepsilon_1},\ \left(\lambda^2 = -\frac{\alpha\pi^2}{L^2}\right);\ \varepsilon_1 = \frac{\pi^2\hbar^2}{2\mu L^2}, \alpha > 1,$$

and performing integration in (5), it is easy to see that the sequence $f(z)=\int \lambda u(z)\,dz$ will converge only if the following condition is performed

$$\frac{\eta}{\alpha}=\frac{FL}{E}<1 \qquad (11)$$

As one can see, the solution of the Schrodinger equation (3) can be written as the Exp. (5) for all the values of the particle energy, which are located in higher values than

$$E_{0i}=FL;\ i=1,2,3,...;\ E_n>E_{0i};\ n=1,2,3,... \qquad (12)$$

i.e. for all those cases where the "coordinate drop" of the particles potential energy $U(z=L)-U(z=0)=FL$ is less than its total energy in the well . Considering the above, for the wave function (5) we obtain the following equation in general form:

$$\psi(t)=C\sin\left\{\pi\sqrt{\alpha}\left[\left(1+\frac{5\eta^2}{32\pi^2\alpha^3}\right)t-\left(\frac{\eta}{4\alpha}-\frac{25\eta^3}{128\pi^2\alpha^4}\right)t^2-\frac{\eta^2t^3}{24\alpha^2}+...\right]\right\}$$
$$\times\exp\left[\frac{\eta}{4\alpha}\left(1-\frac{15\eta^2}{16\pi^2\alpha^3}\right)t+\frac{\eta^2t^2}{8\alpha^2}+\frac{\eta^3t^3}{12\alpha^3}+\frac{\eta^4t^4}{16\alpha^4}+...\right];\left(t=\frac{z}{L}\right), \qquad (13)$$

where C is the normalization constant.

Neglecting the terms of higher order of smallness, and taking into consideration the boundary conditions of the equation (3), we will obtain the following equation for α:

$$\alpha^3-\left(\eta+n^2\right)\alpha^2+\left(\frac{n^2\eta}{2}+\frac{7\eta^2}{24}\right)\alpha-\frac{\eta^2}{16}\left(n^2+\frac{\eta}{2}-\frac{5}{\pi^2}\right)=0;\ (n=1,2,3,...) \qquad (14)$$

The physical solution of this cubic equation [18] has the following form:

$$\alpha_n=\frac{n^2+\eta}{3}\left[1+2\sqrt{1-\frac{12\eta n^2+7\eta^2}{8\left(n^2+\eta\right)^2}}\cos\frac{\varphi(n,\eta)}{3}\right];$$

$$\varphi(n,\eta)=\arccos\left\{\frac{2\left(\frac{\eta+n^2}{3}\right)^3-\frac{\eta+n^2}{3}\left(\frac{\eta n^2}{2}+\frac{7\eta^2}{24}\right)+\frac{\eta^2}{16}\left(n^2+\frac{\eta}{2}-\frac{5}{\pi^2}\right)}{2\left[\frac{\left(\eta+n^2\right)^2}{9}-\frac{\eta}{6}\left(n^2+\frac{7\eta}{12}\right)\right]^{\frac{3}{2}}}\right\} \qquad (15)$$

Accordingly, we obtain the following expression for the envelope wave functions:

$$\psi_{n_c}\left(\eta_c,\alpha_{n_c},t\right)=C_{n_c}\sin\left\{\frac{\pi\alpha_{n_c}^{\frac{3}{2}}}{\eta_c}\left[1-(1-\frac{\eta_c t}{2\alpha_{n_c}})^2\right]\right\}\left(1-\frac{\eta_c t}{2\alpha_{n_c}}\right)^{-\frac{1}{2}} \qquad (16)$$

for the electronic states in the conduction band (c), and

$$\psi_{n_v}\left(\eta_v,\alpha_{n_v},\tau\right)=C_{n_v}\sin\left\{\frac{\pi\alpha_{n_v}^{\frac{3}{2}}}{\eta_v}\left[1-(1-\frac{\eta_v\tau}{2\alpha_{n_{vc}}})^2\right]\right\}\left(1-\frac{\eta_v\tau}{2\alpha_{n_v}}\right)^{-\frac{1}{2}};(\tau=1-t) \tag{16'}$$

for the hole states in the valence band (v). The normalization constants C_{n_c}, C_{n_v} are also calculated exactly and explicitly. Note that for the determination of the hole states we must simply replace the negative charge with the positive one in equation (3), and by mass, effective hole mass is implied. After the change of variable $L-z\rightarrow z'$ we will again arrive at the equation (4).

3. DISCUSSION OF RESULTS

3.1. The spectrum and wave functions

It is known that the solution of equation (3) in the general case [8-9] is given by a linear combination of Airy function at first $Ai(-\xi)$ and second order $Bi(-\xi)$:

$$\psi(\xi)=C_1Ai(-\xi)+C_2Bi(-\xi);\ \xi=\left(z+\frac{E}{F}\right)\left(\frac{2\mu F}{\hbar^2}\right)^{\frac{1}{3}}$$

In this case the energy spectrum of charge carriers in the film will be determined from the relation

$$\frac{Ai(-\xi_0)}{Bi(-\xi_0)}=\frac{Ai(-\xi_L)}{Bi(-\xi_L)}\ , \tag{17}$$

where ξ_0 и ξ_L are the values of the variable ξ in $z = 0$ and $z = L$, respectively.

However, as noted above, both the energy spectrum of carriers, and their wave functions in this case are defined only by numerical methods. The approach offered in this work implies the only limitation for the external field in the form of a condition (11), performing which we get explicit expressions for the spectrum and envelope wave functions of charge carriers for corresponding filed values. It's easy to see, that in the limiting case of weak fields, for $\eta\neq 0$, but $\eta<<1,\ (FL<<\varepsilon_1)$, from the expression (15) we obtain for α_n:

$$\alpha_n=n^2+\frac{\eta}{2}+\frac{\eta^2}{48n^2}\left(1-\frac{15}{\pi^2n^2}\right) \tag{18}$$

which corresponds to the well-known result, when the influence of the external field is taken into account by perturbation theory [8,19]. From the expressions (13) - (16) it's easy to get, that in case of $\eta\rightarrow 0,(F\rightarrow 0)$ $\alpha_n\rightarrow n^2$, and the wave function (16) passes into the wave function of a particle in an infinitely deep square potential well in the absence of the field.

For the range of values of the external field $FL<E_{01}$ equation (14) will determine all the energy levels of particle: $E_1,E_2,E_3,...$, When the field varies in the range $E_{01}<FL<E_{02}$, the equation (14) will now determine the levels $E_2,E_3,E_4,...$, and the level E_1 will here appear lower than the meaning E_{01}, – in the triangular part of

well, produced by an external field, etc. The comparison shows, that the results obtained in the framework of the proposed analytical approach, with sufficient accuracy coincide with the results of the numerical solution of equation (3).

Table1 gives the values of the first three roots of equations (3) and (14), respectively, for intervals of values of the external field $FL < E_{01}, (\eta_{01} = E_{01}/\varepsilon_1)$; As seen from the table, for the states, where $E_n > FL$, the obtained analytical results are with sufficient accuracy close to the data obtained by numerical solution of equation (3). As for the states in the triangular part of the well, when $E_n < FL$, then, taking into account that size quantization is most pronounced for not highly excited states for the analytical description of the first two states in the triangular well, the variation approach, stated in the work [20] can be used.

TABLE 1. α_n values in the range of field values $\eta \in [0, \eta_{01}], \eta_{01} = 1,92067$ for n=1,2,3

η / α_n		0,1	0,3	0,5	0,9	1,4	1,8
$n=1$ α_1	nmr	1,049891	1,149025	1,247294	1,441242	1,678855	1,865133
	anl	1,049906	1,149102	1,247801	1,445313	1,689611	1,876104
$n=2$ α_2	nmr	4,050032	4,1502904	4,250806	4,452602	4,706251	4,9102496
	anl	4,050033	4,150297	4,250895	4,452769	4,706856	4,911502
$n=3$ α_3	nmr	9,050019	9,150173	9,250481	9,451558	9,703768	9,906226
	anl	9,050019	9,150174	9,250484	9,451572	9,703719	9,906331

Note: nmr - numerical result, anl - analytic result

Fig.1 presents the curves $y_{1c}(\eta_c, \alpha_{1c}, t) = \frac{L}{2}|\psi_{1c}(\eta_c, \alpha_{1c}, t)|^2$ for the different values of the external field.

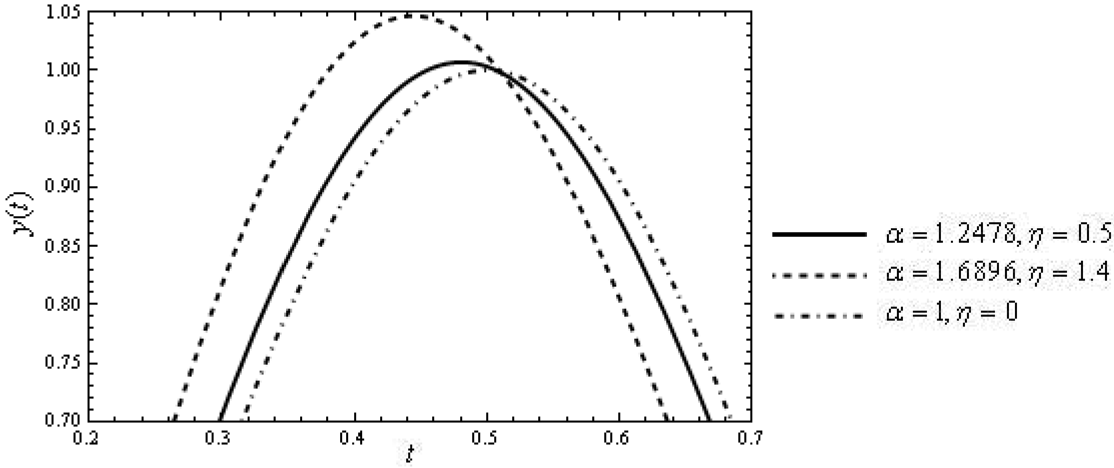

FIGURE 1. Distribution of the probability density $y_{1c}(\eta_c, \alpha_{1c}, t) = \frac{L}{2}|\psi_{1c}(\eta_c, \alpha_{1c}, t)|^2$ for the ground state in the c-band for different values of the external field.

The behavior of the curves shows that with the increase of the field the region of the localization of particles is narrowing in the film along the direction of quantization, in opposite directions for dissimilar charges - and simultaneously increases the amplitude of the corresponding probability density of the spatial distribution of carriers.

3.2. Intersubband electro-optical transitions

As an application of our results we consider the intersubband optical transitions in conduction band the film in the presence of an external field, when $FL < E_{01}$.

Assume that the incident light wave

$$\vec{A}(\vec{r}, t) = \vec{e}A_0 \exp i(\omega t - \vec{q}\vec{r}) + \text{c.c.} \tag{19}$$

with amplitude A_0, frequency ω, wave vector $\vec{q}$ and the single polarization vector $\vec{e}$ directed along the y-axis and linearly polarized along the z axis: $\vec{q} = \vec{q}(0, q, 0), \vec{e} = \vec{e}(0, 0, 1)$.

The corresponding perturbation associated with a weak wave, is represented, as usual [21], in the form

$$\text{A} = \frac{i\hbar|e|}{m_0 c}(\vec{A}\vec{P}) \tag{20}$$

where $\vec{P}-$ is the three-dimensional momentum operator, $m_0 -$ is the free electron mass, e is its charge, $c -$ is the speed of light in vacuum.

For the matrix element $M_{f,i}$ of transitions from the subband $|n_i\rangle$ to subband $|n_f\rangle$ of the conduction band, in general terms one can write [21-22]:

$$M_{f,i} = \frac{i\hbar|e|A_0}{m_0 c} \int_0^L \psi^*_{n_f}\left(\eta_c, \alpha_{n_f}, z\right) \frac{d}{dz} \psi_{n_i}\left(\eta_c, \alpha_{n_i}, z\right) dz \quad (21)$$

Analysis of the expression (21) shows that the external field removes the restrictions on the quantum-sized numbers of subbands n_f, n_i, which are known to occur in the absence of the field [8,22]: $M_{f,i}|_{F=0} \neq 0$, when $n_f \pm n_i = 1,3,5,...$, and $M_{f,i}|_{F=0} = 0$, when $n_f \pm n_i = 2,4,6,...$ When $F \neq 0, M_{f,i}|_{F\neq 0} \neq 0$ for arbitrary $n_f \neq n_i$. As one can see from . Figs. 2, 3, the transitions when $n_f \pm n_i = 1,3,5,...$ stay dominant as formerly: their intensity is a two order of magnitude greater than the intensity of the transitions when $n_f \pm n_i = 2,4,6,...$. With increasing of the field, the intensity of transitions is increasing also.

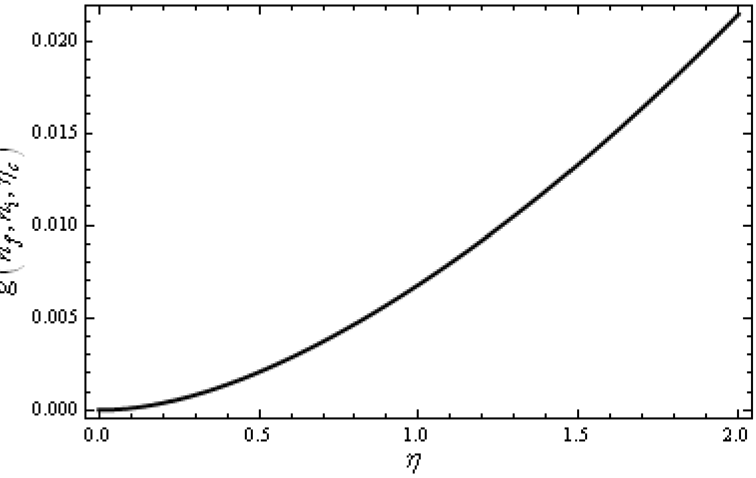

FIGURE 2. The dependence $g\left(n_f, n_i, \eta_c\right) = \left|M_{f,i}\left(n_f, n_i, \eta_c\right)\right|^2 \Big/ \left|\frac{ie\hbar A_0}{m_0 c L}\right|^2$ on the value of external field for the transitions $n_i = 1,\ n_f = 2$.

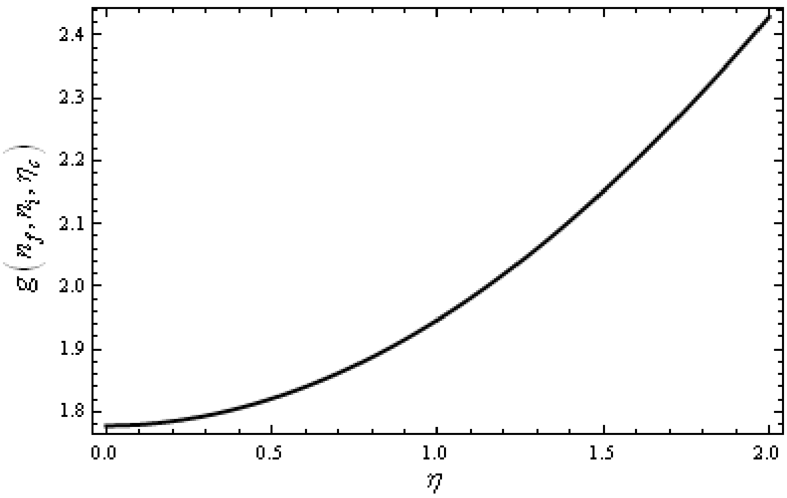

FIGURE 3. The dependence $g\left(n_f, n_i, \eta_c\right) = \left|M_{f,i}\left(n_f, n_i, \eta_c\right)\right|^2 \Big/ \left|\frac{ie\hbar A_0}{m_0 c L}\right|^2$ on the value of external field for the transitions $n_i = 1,\ n_f = 3$.

4. CONCLUSION

Regarding the results obtained in this paper one can conclude the following:
The proposed approach allows us to define explicitly the energy spectrum and envelope wave functions of quantum - confined states of charge carriers for a finite interval of values (from zero) of the external field, which is determined by the parameters of the quantum well film.
The explicit dependence of the electro-optical interband transitions on the relationship between the effective masses of charge carriers opens up the possibility for experimental determination of the value of the effective mass of the carriers.
The obtained analytical results allow for a predictable selection of quantitative changes in the intervals of the external field and the geometric parameters of the sample, which will allow both to change in a controlled manner the optical energy parameters of the system and to regulate the the probability of recombination of opposite charge carriers in the film.

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Formalism of High-Order Derivatives in Quantum Mechanics

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Abstract. A Newtonian mechanics model is essentially the model of a point body in an inertial reference frame. How to describe extended bodies in non-inertial (vibration) reference frames with the random initial conditions? One of the most generalized ways of descriptions (known as the higher derivatives formalism) consists in taking into account the infinitive number of the higher temporal derivatives of the coordinates in the Lagrange function. Such formalism describing physical objects in the infinitive dimensions space does not contradict to the quantum mechanics and infinitive dimensions Hilbert space.

Keywords: Extended Mechanics, high order derivative, vibration reference frame.
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A Newtonian mechanics model is essentially the model of a point body in an inertial reference frame with the defined initial conditions. A point body or an inertial reference frame are hard, if not impossible, to actually find. How one can describe extended bodies in non-inertial reference frames with the random initial conditions? In such a case, Newton's laws would not hold, and to apply them we will have to introduce fictious inertial forces and body center of inertia. In other words, to remain in the framework of the Newtonian model for extended bodies in non-inertial reference frames, we shall have to complement Newton's laws with fictious concepts of the inertia force and the mass center. The latter ones are imaginary physical quantities required only for the sake of remaining in the framework of the Newtonian model.

Abandonment of the Newtonian model and of introduction of the fictious quantities entails a dramatic complicacy in the description of the mechanical system dynamics. The Second Newton's law and all basic equations of the classical mechanics are the second power differential equations. These are complemented with the fictious quantity equations, e.g., equations for fictious forces and the mass center. This yields systems of equations. Such a system of equations may be replaced with a single equation, however, of an order exceeding two. Is there a description of body dynamics with higher-order equations? Yes,

there is, but this is no more Newtonian mechanics. On the other hand, this eliminates the requirement to introduce fictious physical quantities the values of which are deduced indirectly, by calculations using the experimentally measured physical quantities. We shall call the imaginary quantities the fictious ones.
An example of higher-order kinematic characteristics is the case of harmonic oscillations. For this case, there exists an infinite series of higher derivatives. An example of the body dynamics description is the case of harmonic oscillations in an accelerated reference system. An example of a reference system with a free observed body possessing additional kinematic characteristics in the form of higher derivatives is a free body in an oscillating reference system, as well as in a stochastic reference system, that is, a reference system undergoing stochastic oscillations.

In quantum mechanics, such an indirect fictious physical quantity is the wave function. It cannot be directly measured in an experiment, however, it may be employed to calculate values of the quantities observable in an experiment. The quantum mechanics axiomatic comprises the postulate on correspondence of an observable physical quantity to a wave function. Whether the wave function corresponds to a single particle or to multiple particles is still not established. It is also not clear yet how comprehensively does the wave function describe the microobjects. If the quantum mechanics is incomplete, could the quantum mechanical description be complemented with hidden parameters? Speaking generally, any theory is incomplete and requires complementation (and this process is infinite), and what are hidden parameters and what is their physical interpretation is yet unknown.

The equation describing the body dynamics in a stochastic reference system is as follows:

$$F = ma + k_2\ddot{a} + ...$$

For a free particle in the absence of forces this yields a stochastic oscillation equation:

$$ma + k_2\ddot{a} + ... = 0$$

The equation for particle oscillations under the influence of an elastic force in a stochastic reference system takes on the form:

$$k_0 x + ma + k_2\ddot{a} + ... = 0$$

Here we consider the case of systems without friction, braking, and radiation, therefore, we use only even derivatives. There are cases in physics where the oscillation frequencies take on a multitude of discrete values. For example, such is the case of a quantum oscillator $\omega(n + 1/2) = E/\hbar$.

Higher derivatives may be non-local hidden parameters if they describe acceleration and its deduced kinematic parameters in a non-inertial reference system. Then, in any point of the non-inertial system these kinematic parameters shall be identical. If we understand inertial reference systems as those where the Newton's laws hold without introduction of fictious inertia forces, then non-inertial ones are those possessing an acceleration or its derivatives with respect to inertial systems.

Then, stochastic reference systems are non-inertial ones, as they experience oscillations with respect to inertial reference systems. At that, any body free of forces will experience, in a stochastic reference system, stochastic oscillations with kinematic characteristics governed by the reference system. In a non-inertial reference systems kinematic characteristics (acceleration and its derivatives) of a body free of interactions with other bodies shall be governed by kinematic characteristics of the non-inertial reference system with respect to inertial reference systems. For example, in a system uniformly accelerated with respect to inertial reference systems a body free of interactions with other bodies maintains its acceleration, and so on. Therefore, let us introduce the definition:

Inertial reference frames are those where the Newton's laws hold without introduction of the fictious inertia forces.

Abandoning the Newtonian model of a point body in an inertial reference system and considering extended bodies in non-inertial reference frames, let us introduce three :

1. *In an arbitrary reference frame a mass center of an extended body free of interactions with other bodies preserves its kinematic characteristics (velocity, acceleration or their higher derivatives) determined by a constant kinematic characteristics of this observer reference frame with respect to inertial reference frames.*
2. *In an arbitrary reference frame possessing constant kinematic characteristics (velocity, acceleration or their higher derivatives) with respect to inertial reference frames in the form of n derivative of the coordinate in time, the dynamics of the mass center of an extended body under the influence of a force is described by a differential equation of the order 2n:*

$$\alpha_{2n}\dot{q}^{(2n)} + \alpha_{2n-1}\dot{q}^{(2n-1)} + ... + \alpha_2\ddot{q} + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, \dddot{q}, ..., \dot{q}^{(n)}) .$$

α_n being certain constants. Odd derivatives correspond to friction, radiation (losses) or to the case of an open system, that is, for non-bounded and non-isolated systems with external forces (this case corresponds to time irreversibility). The no-loss case is described by the equation

$$\alpha_{n+1}\dot{q}^{(2n)} + ... + \alpha_2\ddot{q} + \alpha_0 q = F(t, q, \dot{q}, \ddot{q}, \dddot{q}, ..., \dot{q}^{(n)}) .$$

If the Galileo's law is generalized for the case of arbitrary reference systems, the invariability of the dynamics laws in this case should mean that in any reference system with n-th order invariant the particle dynamics should be described by a differential equation of the order 2n. The generalized interpretation of Galileo's laws for uniformly accelerated reference systems means that in any reference system uniformly accelerated with respect to inertial reference systems the dynamic of the mass center of the extended body is described by a fourth order differential equation, and so on for higher order derivatives.

3. *In an arbitrary reference system possessing a constant kinematic characteristics (velocity, acceleration or their higher derivatives) with respect to inertial reference systems in the form of n-th derivative of the*

coordinate in time, the forces of interaction of immobile extended bodies do equilibrate.

Let us check whether higher derivatives and functions of higher derivatives relating to fictious quantities in non-inertial reference systems and equivalent to these quantities (according to the equivalency principle) be non-local hidden parameters. For this purpose let us consider quantum micro-objects in the curved space created in non-inertial reference systems.

This approach enables comparison of inertial fictious quantities emerging in stochastic non-inertial reference systems (including gravity fields and waves equivalent to them according to the equivalence principle) with quantum wave properties of micro-objects. This allows posing a problem on relation of this approach with the negative result of experimental detection of gravity waves. Indeed, in such experiments quantum wave effects are not considered wave inertial and wave gravity ones, and what is meant to be detected is actually rejected. Verification of this viewpoint requires new experiments in comparison of quantum wave and gravity inertial properties of micro-objects.

The known descriptions of classical objects (these are classical Newtonian mechanics, Hamilton formalism, Lagrange formalism) and quantum ones (Schrodinger's quantum mechanics, matrix quantum mechanics, etc.) may be complemented with higher derivative formalism. The latter one is capable, without contradicting commonly adopted theories, of complementing the classical descriptions with higher temporal coordinate derivatives in the form of quantum hidden parameters. Higher derivatives may complement both classical and quantum descriptions of physical realm as non-local hidden parameters. The higher derivatives may be regarded as stochastic non-local hidden parameters, complementing classical and quantum descriptions of the physical realm. This allows considering the higher derivative formalism as a stochastic model of the classical mechanics with a transition to stochastic quantum mechanics.

Attempts to build a unified theory for both quantum and classical mechanics are natural and make sense. However, constructing a theory without an axiomatic consistent with both theories resembles a construction without foundation. The point is that the systems of axioms of classical and quantum theories are mutually incompatible and even contradictory. For example, a natural question arises: ”Can the phase space be used, and can the momentum and the coordinate exist simultaneously in quantum mechanics?” The Heisenberg uncertainty principle tells us that it is not possible.

As the quantum theory describes objects in Hilbert space, i.e. in terms of an infinite number of variables, it gives a more detailed description as compared to classical theory. The classical description of physical reality contains an incomparably fewer number of variables. This raises the question: ”How the classical description can be completed?” While a possibility of supplementing the quantum mechanical description with additional hidden variables has been debated for long, the question as to how to complete the classical description to make it

compatible with the quantum mechanical one has not received due attention. The united theory could be based on the recognition of the following postulates:
1. Any reference frame is subject to random external influences, hence every frame is non-inertial and vibrational due to random gravitational fields and waves. Hence, every reference frame is different and a transition from one to another one may lead to jump like changes. The notion of the inertial frame in classical mechanics is valid only on the average and hence the Galilean relativity itself is an average notion.
2. There are many trajectories of a particle corresponding to different reference frames; the Heisenberg uncertainty can be understood as a consequence of the nonexistence of ideal inertial frames in really where the coordinates and momenta can be measure. The Ehrenfest theorem can be seen as a consequence of the inertial frame being an average notion. The ideal inertial frames are non-existent, we can consider the averaging of the classical equations of motion over a time interval Δt:

$$\frac{\partial U}{\partial q} = \frac{d}{dt}\frac{p(t+\Delta t)+p(t-\Delta t)}{2}$$

Using the Taylor expansion

$$p(t\pm\Delta t) = p(t) \pm \dot{p}(t)\Delta t + \frac{1}{2!}\ddot{p}(t)\Delta t^2 + \ldots + \frac{1}{n!}(\pm 1)^n \dot{p}^{(n)}(t)\Delta t^n + \ldots$$

the function $F = -\frac{\partial U}{\partial q}$ can be expanded as follows:

$$F(q,\dot{q},\ddot{q},\dddot{q},\ldots) = \dot{p}(t) + \frac{1}{2!}\dddot{p}(t)\Delta t^2 + \frac{1}{4!}\dot{p}^{(5)}(t)\Delta t^4 + \ldots$$

where $\dot{p}^{(n)}$ denotes n-th time derivative of momentum p.

Correspondingly, the free body preserves the same order of its time derivative like the constant kinematic characteristics of the reference frames. For example, in the uniformly accelerating reference frame the free body preserves its acceleration.
3. The de-Broigle waves $\psi = \psi_0 \exp(-iS/\hbar)$ with the actions functions $S = S(q,\dot{q},\ddot{q},\dddot{q},\ldots,\dot{q}^{(n)},\ldots)$ can be considered as having the gravity-intertial nature following from the fact that every reference frame is vibrational due to the influence of random gravitational fields and waves so that every free particle appears to be oscillating.
4. As the action function $S = S(q,\dot{q},\ddot{q},\dddot{q},\ldots,\dot{q}^{(n)},\ldots)$ is a convergent series in high derivatives of q the difference $S(q,\dot{q},\ddot{q},\dddot{q},\ldots,\dot{q}^{(n)},\ldots) - S(q,\dot{q}) = h$ is finite and can be identified with the constant h. Within the presented framework the variables of the (high order extension of the) phase space do describe the completed dynamics of a particle, but they cannot be measured because the ideal inertial reference frames do not exist in really. The infinite dimensionality of Hilbert space can also be understood as a consequence of all high order time derivatives being taken into account in the description of the dynamics.

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On the fine structure constant in the Machian universe

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Abstract

Within the Machian model of the universe the dark energy is identified with the energy of collective gravitational interactions of all particles inside the Hubble horizon. It is shown that the fine structure constant can be expressed in terms of the observed radiation, baryon and dark energy densities of the universe and the densities of various components of matter are interrelated via it.

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Keywords: Thermodynamic gravity, dark energy, fine structure constant

There exist three types of field theoretical descriptions successfully working in their own areas: quantum mechanics, quantum field theory, and general relativity. However, problems arise, when one tries to establish bridges between these descriptions. There are well known difficulties in the foundations of relativistic quantum mechanics, the definition of quantum fields in curved space-times, and quantization of gravity.

A new framework to establish a unifying approach to main field theoretical descriptions is provided by the approach appealing to the notions of thermodynamics. For instance, the thermodynamic arguments and analogies have proven to be useful in black hole physics [1], discovery of Unruh temperature [2], establishment of the AdS/CFT correspondence [3], derivation of the Einstein [4] and Maxwell [5] equations, in the recent attempt to interpret the Newtonian gravity as an entropic force [6], and other discussions of the "emergent gravity" [7,8]. Besides, the analogies with the classical statistical mechanics and thermodynamics have been underlying some recent discussions of the foundations of quantum mechanics [9,10].

However, these thermodynamic arguments are mere analogies so far, since the idea that classical field theory is a consequence of the laws of thermodynamics and statistical physics raises many questions, such as the origin of the microscopic degrees of freedom responsible for the thermodynamics-like quantities appearing in the formalism and the origin of the relativistic invariance. The latter is due to the fact that the approaches based on statistical thermodynamics imply the existence of the energy reservoir of the canonical ensemble which singles out a preferred reference frame where the average momentum of the ensemble is zero, so that the underlying theory is not even Galilean invariant.

A Machian thermodynamic model of the universe has been put forward in [11-14] as a possible answer to the problems outlined above. In this model the universe is viewed as the statistical ensemble of all quantum particles inside the horizon. The 'thermodynamic' formulation of the Mach's principle is introduced (for other formulations see [15]) according to which the rest mass of a particle is a measure of its long-range collective interactions with the whole statistical ensemble of particles inside the cosmological horizon. The ensemble distinguishes a fundamental cosmological reference frame. In spite of the explicitly anti-relativistic assertion above, it was demonstrated how the model can be compatible with the existing cosmological and gravitational theories in the low energy regime [12,13], where the relativistic invariance emerges, and with the main features of quantum mechanics [14]. Note that the assumption that all particles in the isotropic and homogeneous universe are involved in mutual long-range gravitational interactions avoids the problem with the anisotropic effective masses which arose in earlier naive treatments of the Mach's principle and have been ruled out by the precision measurements [16]. Besides, this assumption effectively weakens the observed strength of gravity, thus explaining the hierarchy problem in particle physics [11].

In this paper we show that estimations based on the Machian thermodynamic model of the universe lead to the expression of the fine structure constant in terms of the observable cosmological parameters.

In the model [11-14] each particle in the universe interacts with the collective gravitational potential of all N particles in the universe. The universe is considered as a statistical ensemble of 'gravitationally entangled' particles, where the collective non-local (at the Hubble scale) interaction gives rise to what we usually perceive as classical space-time. The universal constant of the speed of light, c, originates in the non-local potential of the whole universe, Φ, acting on each particle of the world ensemble:

$$c^2 = - \Phi = 2MG/R \,, \tag{1}$$

where M and $R \approx 3\times10^{26}$ m are the total mass and the radius of the universe, respectively, and $G \approx 7\times10^{-11}$ $m^3s^{-2}kg^{-1}$ is the Newton constant. This 'universal' potential Φ, and thus c, can be regarded as constants since, according to the cosmological principle, the universe is isotropic and homogeneous at the Hubble scale R.

Let us emphasize that (1) is just another formulation of the critical density condition in relativistic cosmology:

$$\rho_c = 3M/4\pi R^3 = 3H^2/8\pi G \, , \tag{2}$$

where

$$H = c/R \approx 2\times10^{-18} \text{ s}^{-1} \tag{3}$$

is the Hubble constant. The later relation, which relates the Hubble radius of the universe R to the distance the light has traveled since the big bang, is regarded only as a coincidence in *LCDM* model where this observed relation is valid only once in the history of the universe, namely in the present epoch. However, if we impose the condition (1) the Friedmann equations lead to the unique solution corresponding to (3).

The relation (1) also allows us to formulate the Mach's principle which relates the origin of inertia of a particle, or its rest energy, to the particle's interactions with the whole universe, namely

$$E = mc^2 = - m\Phi \, , \tag{4}$$

where m is the mass parameter describing the particle's inertia. This equation is equivalent to

$$mc^2 + m\Phi = 0 \, , \tag{5}$$

and represents the simplest energy balance equation written here for a particle at rest with respect to the preferred reference frame of the universe. The energy balance conditions, which exhibit the exact conservation of energy [12,13], assume that the non-local gravitational interaction with the universe is the source of all kinds of local energy of a particle. This means that the total (nonlocal plus local) energy of any object in the universe vanishes, i.e. the gravitational energy is assumed to be negative, while all other forms of energy are positive [17]. In the homogeneous and isotropic universe this statement can be generalized to any expanding region with the radius r and the energy density ρ. Indeed, it can be shown that the Friedmann equation for the region is equivalent to the classical energy conservation condition and the curvature constant k entering this equation can be related to the total Newtonian (kinetic plus potential) energy E_{tot} [18]:

$$k = -\,5E_{tot}/2\pi r^5 a^3 \rho\,, \tag{6}$$

where a is the cosmological scale factor. The current cosmological measurements (see, e.g. [19]) are indicating that most likely $k = 0$. Hence, the total energy of any region, and thus of the whole universe, vanishes. Since in this case the universe can emerge without violation of the energy conservation, this point of view appears to be preferable in cosmology [17,20].

The potential Φ takes into account the contribution of the collective gravitational interactions between all N particles inside the horizon. Namely, since each particle interacts with all other $(N-1)$ particles, and the mean separation in the interacting pairs is $R/2$, the total energy consists of $N(N-1)/2$ terms of magnitude $\approx 2Gm^2/R$. Then, for very large N, the energy of a single particle which interacts with the total gravitational potential of the universe Φ is given by:

$$E \approx N^2 Gm^2/R\,. \tag{7}$$

Correspondingly, the contribution of the collective gravitational interactions to the total mass of the universe is:

$$M_G \approx N^2 m/2\,, \tag{8}$$

so that $M_G \sim M$, where M denotes the total mass of the universe, is of the order of N^2 and not $\approx Nm$. The numerical value of the total mass of the universe can be estimated from (2) or (1) by using the observed values of c, G and H:

$$M \sim c^3/2GH \approx 10^{53}\ kg\,. \tag{9}$$

Because of the finite number of particles inside the horizon and the existence of the maximal speed c, any movement of the particles of the 'gravitationally entangled' world ensemble results in a delayed response of the whole ensemble. The response time of the universe to the motion of a quantum particle is estimated as follows:

$$\Delta t \sim R/Nc \sim 1/NH\,. \tag{10}$$

Note that Δt is much shorter than e.g. the mean free motion time of particles with the mean separation $\sim R/N^{1/3}$ in a dilute gas, because, as a result of the non-local interactions of all particles, the effective mean separation between particles in the world ensemble is much shorter: $\sim R/N$.

As a consequence of the delayed nonlocal response of the universe, any mechanical process in the world ensemble will be accompanied by the exchange of at least the minimal amount of action $A = mc^2\Delta t$, which we identify with the Planck's action quantum [12,13]:

$$A = -\int dt\, E \approx - mc^2 \Delta t = 2\pi\hbar \,. \tag{11}$$

Using (9), (10) and (11) we can estimate the total action of the universe:

$$A_U = Mc^2/H \approx N^3 A/2 \,, \tag{12}$$

and the number of typical particles in it:

$$N \approx (2A_U/A)^{1/3} \approx (Mc^2/\pi\hbar H)^{1/3} \approx 10^{40} \,. \tag{13}$$

This number, one of the main parameters of our model, is known to have appeared in a different context in the Dirac's 'large numbers' hypothesis which is usually considered as an indication of a deep connection between the micro and macro physics [21].

Using (8), (9), (13), and the assumption $M_G \sim M$, we can also estimate the mass of a typical particle in our simplified model universe:

$$m \approx 2M/N^2 \approx 2\times 10^{-27}\ kg \approx 1\ GeV\ c^{-2} \,, \tag{14}$$

which appears to be of the order of magnitude of the proton mass. This estimation is also consistent with (10), as

$$\Delta t \approx 1/NH \approx 10^{-22}\ s \sim \hbar\ GeV^{-1} \,, \tag{15}$$

Hence, a typical stable heavy particle, the proton, can be considered as a typical particle forming the gravitating world ensemble in our simplified one-component universe. Or, vice versa, we could postulate that the typical mass of a particle forming the world ensemble equals to the proton mass (i.e. a typical stable baryon), and then use (10) in order to obtain the mean action per particle (11), which then exactly coincides with the Planck constant.

Let us stress here a difference between our model and the standard approach, where the mass of the universe (9) is estimated as the sum of masses of $\sim 10^{80}$ protons. In the thermodynamic model the energy of long-range interactions of all particles is taken into account, which is missing in the standard approach. As a consequence, according to eq. (8), only $\sim 10^{40}$ protons, eq. (13), is sufficient in order to account for the correct value of the total mass of the universe, eq. (9). For this reason, it is natural to identify the collective gravitational energy of all particles with the dark energy of the universe which, within the standard *LCDM* model, is identified with the cosmological constant Λ. Thus, we assume:

$$\Omega_\Lambda = M_G/M \approx N^2 m/2M \,. \tag{16}$$

Note that under the above identification the energy balance condition (5) written for all N particles in the universe is equivalent to the equation of state of the dark energy in the standard cosmology:

$$\rho + p = 0\,, \tag{17}$$

where ρ is the energy density. In our case the exotic negative pressure p has a natural explanation: it is a consequence of the negative gravitational potential of the whole universe. The assumption (16) can also help to resolve the problem of why the density of dark energy is of the order of the critical density (2). The standard cosmology offers no reasonable explanation of this relationship since the dark energy is associated with the energy of quantum vacuum fluctuations which is expected to be 120 orders of magnitude higher than ρ_c.

Now, let us consider a little bit more realistic model universe which includes both neutral and charged particles. We assume that the universe as a whole is neutral, i.e. a half of charged particles carries positive charge $+e$ and the other half have negative charge $-e$. The number of charged particles can be roughly identified with the number of baryons in the universe $N_b < N$. A simple combinatorics yields for the gravitational energy of single baryon which interacts with all other particles in the universe the following formula:

$$E_b = (2N_bN - N_b^2)Gm^2/R \approx 2N_bNGm^2/R\,. \tag{18}$$

Then according to (8) the total gravitational energy of the baryon component of matter yields:

$$E_{b|G} = N^2E_b/2 \approx N_bN^3Gm^2/R\,. \tag{19}$$

We can also expect that the ratio (16) of the gravitational and total energy is valid for the corresponding baryon contributions:

$$E_{b|G}/E_{b|tot} = \Omega_\Lambda\,, \tag{20}$$

where $E_{b|tot}$ denotes the total energy of the baryon component of the universe. Then the observed baryon density in the universe can be written in the form:

$$\Omega_b \approx (E_{b|tot} - E_{b|G})/Mc^2 \approx E_{b|G}(1 - \Omega_\Lambda)/\Omega_\Lambda Mc^2\,. \tag{21}$$

Further, let us estimate the electromagnetic energy of all N_b charged particles (i.e. $N_b/2$ interacting pairs) in the model universe. The fact that electric charges have two polarities, while the mass is always positive, leads to basic differences, namely, universe as a whole is neutral and, in contrast to the gravitational energy, the total electromagnetic, or radiative energy consists of $N_b/2$ additive terms, i.e.

$$E_r \approx N_b k_c e^2/2R = N_b \alpha\hbar c/2R \,, \quad (22)$$

where k_c is the Coulomb constant and α is the fine structure constant. Similar to (19) the total gravitational energy of the radiation component of matter can be estimated as:

$$E_{r|G} \approx N_b N^2 \alpha\hbar c/4R \,. \quad (23)$$

Equations (19), (20), (21) and (23) yield for the ratio of the radiative and baryon densities in the universe:

$$\Omega_r/\Omega_b \approx E_{r|G}\Omega_\Lambda/E_{b|G}(1 - \Omega_\Lambda) \approx \alpha\hbar c\Omega_\Lambda/4NGm^2(1 - \Omega_\Lambda) \,, \quad (24)$$

whence it follows:

$$\alpha = 4NGm^2\Omega_r(1 - \Omega_\Lambda)/\Omega_\Lambda\Omega_b\hbar c \,. \quad (25)$$

From (1), (7) and (8) we find:

$$m^2/\hbar = 2\pi c\Omega_\Lambda/NG \,. \quad (26)$$

Finally, by inserting this formula into (25), we arrive at the expression for the fine structure constant in terms of the cosmological parameters Ω_Λ, Ω_b and Ω_r:

$$\alpha \approx 8\pi\Omega_r(1 - \Omega_\Lambda)/\Omega_b \,. \quad (27)$$

From the contemporary observations we know that $\Omega_r/\Omega_b = 1.09 \pm 0.03\times10^{-3}$ and $\Omega_\Lambda = 0.74 \pm 0.03$ [22]. Using those values in (27), we obtain:

$$\alpha \approx 7.1 \pm 0.6\times10^{-3} \,, \quad (28)$$

which is surprisingly close to the experimental value $\alpha \approx 7.297\times10^{-3}$.

From (27) we can also obtain a relationship between the dark energy and the baryon energy densities using the observed values of the fine structure constant **α** and the radiation energy density $\Omega_r = 4.8 \pm 0.04\times10^{-5}$ [22]:

$$\Omega_\Lambda \approx 1 - 6\Omega_b \,, \quad (29)$$

which appears to be consistent with current observations. Note that in the standard cosmological models the relation between the baryonic matter and dark energy densities is an arbitrary parameter determined from observations. Using the relationship between the densities of various matter species Ω_i and Ω_Λ in the flat universe:

$$\Omega_\Lambda + \sum_i \Omega_i = 1 , \quad (30)$$

we can exclude Ω_Λ from (27) and interpret the result as a relation between the fine structure constant and the densities of various components of matter in the universe.

To conclude, we have considered the energy content of the Machian thermodynamic model of universe taking into account the existence of charged particles and their contribution to the total gravitational energy. The energy of collective gravitational interactions of all particles in the universe was identified with the dark energy. It has allowed us to express the fine structure constant in terms of the radiation, baryon and dark energy densities. The expression agrees with the experimental value of the fine structure constant up to the uncertainty in the observed values of the cosmological parameters. Being depending on the energy densities of different components of matter in the universe, the expression might be helpful for understanding the recent evidence of the cosmological variations of the fine structure constant at high redshifts [23]. The expression also yields a proportion between the densities of the baryon and dark energy components in the universe and a relation between the densities of various species of matter in the universe in terms on the observed (current) value of the fine structure constant.

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Is Planck's Constant a Cosmological Variable?

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Abstract. Within the expanding cosmic Hubble volume, Hubble length can be considered as the gravitational or electromagnetic interaction range. Product of 'Hubble volume' and 'cosmic critical density' can be called as the 'Hubble mass'. Based on this cosmic mass unit, authors noticed five peculiar semi empirical relations in atomic, nuclear and cosmic physics. With these applications it is possible to say that – during the cosmic evolution, magnitude of Planck's constant increases with increasing cosmic time. This may be the root cause of observed cosmic red shifts. By observing the cosmological rate of change in Planck's constant, the future cosmic acceleration can be verified from the ground based laboratory experiments. With reference to the current concepts of distant and spatial variation of the fine structure ratio, variation of the Planck's constant can be considered for further analysis.

Keywords: Hubble volume; Cosmic critical density; Hubble mass; Cosmic thermal energy density; Planck's constant;
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1. INTRODUCTION

Einstein, more than any other physicist, untroubled by either quantum uncertainty or classical complexity, believed in the possibility of a complete, perhaps final, theory of everything. He also believed that the fundamental laws and principles that would embody such a theory would be simple, powerful and beautiful. Physicists are an ambitious lot, but Einstein was the most ambitious of all. His demands of a fundamental theory were extremely strong. If a theory contained any arbitrary features or undetermined parameters then it was deficient, and the deficiency pointed the way to a deeper and more profound and more predictive theory. There should be no free parameters – no arbitrariness. According to his philosophy, electromagnetism must be unified with general relativity, so that one could not simply imagine that it did not exist. Furthermore, the existence of matter, the mass and the charge of the electron and the proton (the only elementary particles recognized back in the 1920s), were arbitrary features. One of the main goals of a unified theory should be to explain the existence and calculate the properties of matter.

In physics history, for any new idea or observation or new model - at the very beginning – their existence was very doubtful. The best examples were : 1) Existence of atom 2) Existence of quantum of energy 3) Existence of integral nature of angular momentum 4) Existence of wave mechanics 5) Existence of Black holes 6) Black hole radiation and so on. Another best example is/was: Einstein's cosmological λ term. In this paper authors made an attempt to understand the basic concepts of particle cosmology via five semi empirical applications.

If we write $R_0 \cong (c/H_0)$ as a characteristic cosmic Hubble radius [1] then the characteristic cosmic Hubble volume is $V_0 \cong (4\pi/3)R_0^3$. The Hubble volume is sometimes defined as a volume of the universe with a commoving size of (c/H_0). The exact definition varies. Some cosmologists even use the term Hubble volume to refer to the volume of the observable universe. With reference to the cosmic critical density $\rho_c \cong \left(3H_0^2/8\pi G\right)$ and the characteristic cosmic Hubble volume, characteristic cosmic Hubble mass can be expressed as

$$M_0 \cong \rho_c V_0 \cong (c^3/2GH_0) \tag{1}$$

If we do not yet know whether the universe is spatially closed or open, then the idea of Hubble volume or Hubble mass can be used as a tool in cosmology and unification. This idea is very close to the Mach's idea of distance cosmic back ground. It seems to be a quantitative description to the Mach's principle. In understanding the basic concepts of unification of the four cosmological interactions, the cosmic radius (c/H_0) can be considered as the infinite range of the gravitational or electromagnetic interaction.

Within the Hubble volume it is noticed that: 1) Each and every point in free space is influenced by the Hubble mass. 2) Hubble mass plays a vital role in understanding the properties of electromagnetic and nuclear interactions and 3) Hubble mass plays a key role in understanding the geometry of the universe. The current value of the Hubble's constant is $H_0 \cong 70.4^{+1.3}_{-1.4}$ Km/sec/Mpc [2,3]. Thus the magnitude of the present cosmic Hubble mass can be given as $M_0 \cong 8.84811\times10^{52}$ Kg.

2. FIVE PECULIAR APPLICATIONS

Application-1

In physics, the fine-structure ratio (usually denoted by α) is a fundamental physical constant, namely the coupling constant characterizing the strength of the electromagnetic interaction. Being a dimensionless quantity, it has constant numerical value in all systems of units. The most precise value of α obtained experimentally (as of 2012) is based on a measurement of 'Linde g factor' using a 'one-electron' so-called 'quantum cyclotron' apparatus, together with a calculation via the theory of QED. This measurement of α has a precision of 0.25 parts per billion.

If $\rho_c c^2$ is the present cosmic critical energy density and aT_0^4 is the present cosmic thermal energy density, it is noticed that,

$$\ln\sqrt{\frac{aT_0^4}{\rho_c c^2}\cdot\frac{4\pi\varepsilon_0 GM_0^2}{e^2}} \cong \left(\frac{1}{\alpha}\right) \tag{2}$$

This is a very peculiar relation and constitutes the $\rho_c c^2$ and aT_0^4. Note that, from unification point of view, till today role of dark energy or dark matter is unclear and undecided. Their laboratory or physical existence is also not yet confirmed. In this critical situation this application can be considered as a key tool in particle cosmology. Note that large dimensionless constants and compound physical constants reflect an intrinsic property of nature. At present if $\rho_c \cong \left(3H_0^2/8\pi G\right)$, independent of the gravitational constant, Eq.1 takes the following form.

$$\ln\sqrt{\frac{2\pi}{3}\cdot\frac{4\pi\varepsilon_0 aT_0^4 c^4}{e^2 H_0^4}} \cong \frac{1}{\alpha} \tag{3}$$

At present if observed CMBR temperature [4] is $T_0 \cong 2.725\ {}^0\text{K}$, obtained $H_0 \cong 71.415$ Km/sec/Mpc. After simplification, it can be interpreted as follows. Total thermal energy in the present Hubble volume can be expressed as,

$$\left(E_T\right)_0 \cong aT_0^4 \cdot \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 \tag{4}$$

If $\left(\frac{c}{H_0}\right)$ is the present electromagnetic interaction range, then the present electromagnetic potential can be expressed as

$$\left(E_e\right)_0 \cong \frac{e^2}{4\pi\varepsilon_0\left(c/H_0\right)} \tag{5}$$

Now inverse of the present fine structure ratio can be expressed as

$$\left(\frac{1}{\alpha}\right)_0 \cong \ln\sqrt{\frac{\left(E_T\right)_0}{2\left(E_e\right)_0}} \tag{6}$$

In this way, in a unified manner, the present fine structure ratio can be fitted. From this relation it is possible to say that, cosmological rate of change in fine structure ratio, $(d\alpha/dt)$ may be considered as an index of the future cosmic acceleration. Many physicists think its possible variation and experiments are in progress. Dirac proposed about the variation of the gravitational constant [6,7]. Compared to the concept of 'variation of gravitational constant' – 'variation of fine structure ratio' seems to be testable in ground based spectroscopic observations easily. While the fine-structure constant is known to approach 1/128 at interaction energies above 80

GeV, physicists have pondered for many years whether the fine-structure constant is in fact a constant, i.e., whether or not its value differs by location and over time. Specifically, a varying α has been proposed as a way of solving problems in cosmology and astrophysics.

More recently, theoretical interest in varying constants (not just α) has been motivated by string theory and other such proposals for going beyond the Standard Model of particle physics. The first experimental tests of this question examined the spectral lines of distant astronomical objects and the products of radioactive decay in the Oklo natural nuclear fission reactor. The findings were consistent with no change. In October 2011 Webb *et al.* reported a variation in α dependent on both redshift and spatial direction [8]. Till today from ground based laboratory experiments no variation was noticed in the magnitude of the fine structure ratio. Future experiments and observations may reveal the real picture.

Application-2

If M_p is the Planck mass it is noticed that,

$$\ln\left(\frac{m_e R_0^2}{M_p R_s^2}\right) \cong \frac{1}{\alpha}. \tag{7}$$

where $R_0 \cong (c/H_0)$ and R_s is close to *1.5 fm* and can be considered as the characteristic nuclear radius or the strong interaction range [5]. Interpretation of this relation seems to be connected with two lengths and two mass units. In this semi empirical relation the most puzzling thing is that, out of the 4 physical LHS parameters, 3 are believed to be fundamental physical constants and they are electron rest mass, Planck mass and the strong interaction range. The only variable is Hubble length. In RHS, the output physical constant is the fine structure ratio. Here interpretation seems to be a sensitive and critical task.

Application-3

If m_p and m_e are the rest masses of proton and electron respectively, it is noticed that

$$\frac{G\sqrt{M_0\sqrt{m_p m_e}}}{c^2} \cong 1.38 \text{ fm} \tag{8}$$

This obtained length is close to the strong interaction range [5]. Whether it is the strong interaction range or something else, is not clear. Here in RHS, the coefficient 2 is missing. From unification point of view this relation can be given a chance either in quantum chromodynamics or in string theory. From the above two applications, it is possible to say that, the Hubble length plays a key role in atomic and nuclear physics.

Application-4

Another peculiar relation can be expressed in the following way.

$$\frac{\hbar c}{Gm_p\sqrt{M_0 m_e}} \cong 0.9975 \cong 1 \tag{9}$$

If this is merely a coincidence, it is very good and the matter ends here. This relation seems to be a mysterious and confusing one. This relation can be analyzed in different angles. Its applications seem to be very mysterious. With the above relation, obtained value of the present Hubble's constant is $H_0 \cong 70.75$ Km/sec/Mpc. Now the 'Bohr radius of hydrogen' atom can be expressed as

$$a_0 \cong \left(\frac{4\pi\varepsilon_0 Gm_p M_0}{e^2}\right)\frac{Gm_p}{c^2} \cong \frac{1}{2}\left(\frac{4\pi\varepsilon_0 Gm_p^2}{e^2}\right)\cdot\frac{c}{H_0} \tag{10}$$

This relation is free from the famous constant h(cross). If nuclear mass increases as $(n.m_p)$ where n =1,2,3,.. it is very simple to understand the integral nature of angular momentum. Above relation takes the following form.

$$n^2 a_0 \cong \left(\frac{4\pi\varepsilon_0 G(n\cdot m_p)M_0}{e^2}\right)\frac{G(n\cdot m_p)}{c^2} \cong \frac{1}{2}\left(\frac{4\pi\varepsilon_0 G(n.m_p)^2}{e^2}\right)\cdot\frac{c}{H_0} \tag{11}$$

From all these relations it can be interpreted that, in the presently believed atomic and nuclear "physical constants", there exists one cosmological variable! By observing its cosmological rate of change, the "future" cosmic acceleration can be verified. Thus independent of the cosmic redshift and CMBR observations, with these coincidences it is possible to understand and decide the cosmic geometry. Now in a very simple way, h(cross) and Planck's constant can be expressed as

$$n\cdot\hbar \cong \sqrt{\frac{M_0}{m_e}}\cdot\frac{G(n\cdot m_p)m_e}{c} \tag{12}$$

$$n\cdot h \cong 2\pi\sqrt{\frac{M_0}{m_e}}\cdot\frac{G(n\cdot m_p)m_e}{c} \tag{13}$$

In this way, in a very simplified manner, the integral nature of angular momentum can be understood. This interpretation seems to be quite interesting but at the same time it is very difficult to accept this observation. Considering any two consecutive integers n and $(n+1)$, their geometric mean state can be expressed as $\sqrt{n(n+1)}$ and it seems to be the base for the vector atom model. The fine structure ratio can be expressed as

$$\alpha \cong \sqrt{\frac{m_e}{M_0}}\cdot\frac{e^2}{4\pi\varepsilon_0 Gm_p m_e} \tag{14}$$

Please note that, Einstein never believed in Quantum mechanics [9]. Because of his opposition to quantum mechanics he allowed himself to ignore most of the important developments in fundamental physics for over twenty five years, as he himself admitted in 1954, 'I must seem like an ostrich who buries its head in the relativistic sand in order not to face the evil quanta'. David Gross says [9] : To be sure many of the inventors of quantum field theory were soon to abandon it when faced with ultraviolet divergences, but it is hard to understand how Einstein, could not have been impressed with the successes of the marriage of his children quantum mechanics and special relativity. The Dirac equation and quantum electrodynamics had remarkable successes, especially the prediction of anti-particles. How could Einstein not have been impressed?

After sometime in the late 1920s Einstein became more and more isolated from the mainstream of fundamental physics. To a large extent this was due to his attitude towards quantum mechanics, the field to which he had made so many revolutionary contributions. Einstein, who understood better than most the implications of the emerging interpretations of quantum mechanics, could never accept it as a final theory of physics. He had no doubt that it worked, that it was a successful interim theory of physics, but he was convinced that it would be eventually replaced by a deeper, deterministic theory. His main hope in this regard seems to have been the hope that by demanding singularity free solutions of the nonlinear equations of general relativity one would get an over determined system of equations that would lead to quantization conditions.

Application-5

With reference to the Planck mass

$$M_P \cong \sqrt{\frac{\hbar c}{G}} \tag{15}$$

and the elementary charge e, a new mass unit

$$M_C \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \tag{16}$$

can be constructed. With M_C and M_0 it can be assumed that, cosmic thermal energy density, matter energy density and the critical energy density are in geometric series and the geometric ratio is $1+\ln(M_0/M_C)$. Thus,

$$\left(\frac{\rho_c c^2}{\rho_m c^2}\right)_0 \cong 1+\ln\left(\frac{M_0}{M_C}\right) \cong 143.0 \tag{17}$$

where ρ_m is the cosmic matter density.

$$\left(\frac{\rho_c c^2}{aT^4}\right)_0 \cong \left[1+\ln\left(\frac{M_0}{M_C}\right)\right]^2 \cong (143.0)^2 \tag{18}$$

At present, these relations take the following trial-error form:

$$\left[1+\ln\left(\frac{c^3}{2GH_0M_C}\right)\right]^{-1} H_0 \cong \sqrt{\frac{8\pi G a T_0^4}{3c^2}} \tag{19}$$

From this relation, if T_0 is known, by trial–error, present value of H_0 can be estimated. Note that, obtained matter density ρ_m can be compared with the elliptical and spiral galaxy matter density. Based on the average mass-to-light ratio for any galaxy [10]

$$\left(\rho_m\right)_0 \cong 1.5\times10^{-32}\eta h_0 \;\text{ gram/cm}^3 \tag{20}$$

where for any galaxy, $\langle M/L\rangle_{Galaxy} = \eta\langle M/L\rangle_{\text{Sun}}$ and the number: $h_0 \cong \frac{H_0}{100 \text{ Km/sec/Mpc}} \cong \frac{70.75}{100} \cong 0.7075.$ Note that elliptical galaxies probably comprise about 60% of the galaxies in the universe and spiral galaxies are thought to make up about 20% of the galaxies in the universe. Almost 80% of the galaxies are in the form of elliptical and spiral galaxies. For spiral galaxies, $\eta h_0^{-1} \cong 9 \pm 1$ and for elliptical galaxies, $\eta h_0^{-1} \cong 10 \pm 2$. For our galaxy inner part, $\eta h_0^{-1} \cong 6 \pm 2$. Thus the average ηh_0^{-1} is very close to 8 to 9 and its corresponding matter density is close to (6.0 to 6.67) $\times 10^{-32}$ gram/cm^3.

3. DISCUSSION AND CONCLUSIONS

Hubble initially interpreted red shifts as a Doppler effect, due to the motion of the galaxies as they receded for our location in the Universe. He called it a ‘Doppler effect’ as though the galaxies were moving ‘through space’; that is how some astronomers initially perceived it [1]. This is different to what has now become accepted but observations alone could not distinguish between the two concepts. Later in his life Hubble deviated from his earlier interpretation [11] and said that the Hubble law was due to a hitherto undiscovered mechanism, but not due to expansion of space - now called ‘cosmological expansion’. This is a very important point to be noted here. With reference to the noticed semi empirical relations - the observed cosmic red shifts can be interpreted in the following way:.

1) During the cosmic evolution, the magnitude of Planck’s constant increases and the quantum of energy gradually increases. At present, at all galaxies (either aged or younger), value of Planck’s constant is same. Based on the current concepts of spatial variation of the fine structure ratio, this proposal may be given a chance and may not be ignored.
2) $\frac{d(h)}{dt}$ is a measure of cosmic rate of expansion. It may be noted that, as the universe is accelerating, value of Planck’s constant increases. Thus if there is no change in the magnitude of Planck’s constant, it can be suggested that, at present there is no cosmic acceleration.
3) During journey light quanta will not lose its energy.
4) Past light quanta emitted from aged galaxy will have less Planck’s constant and show a red shift with reference to the receiving younger galaxy.

Every day quantum mechanics is strongly connected with the constancy of Planck's constant. String theory, quantum cosmology, quantum chromodynamics (QCD) etc. are strongly based on the constancy of Planck's constant. With reference to the present concepts of cosmic acceleration and with laboratory experiments one may not decide whether universe is accelerating or decelerating. Many experiments are under progress to detect and confirm the existence of dark matter and dark energy. Along with these experiments if one is willing to think in this new direction, from atomic and nuclear inputs it may be possible to verify the future cosmic acceleration.

With the proposed concepts and with the advancing science and technology, from the ground based laboratory experiments, from time to time the concept $\frac{d(h)}{dt}$ can be put for experimental tests. There is no need to design a new experiment. Well established experiments are already available by which Planck's constant can be estimated. Moreover, conducting an experiment in this direction is also very simple. Only thing is that the same experiment has to be repeated for several times or continuously. This is also very simple. Thus in the near future one can expect the real picture.

Alternatively in a theoretical way, the proposed five applications or semi empirical relations can be given a chance and the subject of elementary particle physics and cosmology can be studied in a unified manner [12]. It is true that the proposed relations are speculative and peculiar also. By using the proposed relations and applying them in fundamental physics, in due course their role or existence can be verified. With these relations, Hubble constant can be estimated from atomic and nuclear physical constants. If one is able to derive them with a suitable mathematical model, independent of the cosmic redshift and CMBR observations, the future cosmic acceleration can be verified from atomic and nuclear physical constants. Now the new set of proposed relations are open to the science community. Whether to consider them or discard them depends on the physical interpretations, logics, experiments and observations. In most of the critical cases, 'time' only will decide the issue. The mystery can be resolved only with further research, analysis, discussions and encouragement.

4. NEW DERIVATION FOR COSMIC RED SHIFT

Let us revise the basic definition of (z) as follows:

$$z \cong \frac{\lambda_0 - \lambda_G}{\lambda_0} \cong \frac{\Delta\lambda}{\lambda_0} \text{ but not } \frac{\Delta\lambda}{\lambda_G} \tag{21}$$

Here λ_0 is the wave length of light at our galaxy and λ_G is the wave length of light at old galaxy. Note that when $\Delta\lambda$ is very small or $\lambda_0 \cong \lambda_G$

$$\frac{\Delta\lambda}{\lambda_0} \cong \frac{\Delta\lambda}{\lambda_G} \tag{22}$$

Please note that, by Hubble's time the observed maximum red shift was 0.003. With that red shift it is not possible to decide the correct definition of z.

Based on the increasing value of the Planck's constant, red shift (z) will be directly proportional to the age difference of our galaxy and the old galaxy (Δt).

$$z \propto \Delta t \tag{23}$$

$$z \cong H_0 \Delta t \tag{24}$$

Here H_0 is the proportionality constant. In this way H_0 can be incorporated directly. When $\Delta t \to 0 \;\; \Rightarrow \;\; \Delta\lambda \to 0 \;\; \Rightarrow z \to 0$ Multiplying Eq. 24 both sides with c

$$zc \cong c\Delta t H_0 \tag{25}$$

On rearranging,

$$c\Delta t \cong z \cdot \frac{c}{H_0} \tag{26}$$

If $c\Delta t$ represents the distance between our galaxy and the distant old galaxy,

$$d \cong c\Delta t \cong z \cdot \frac{c}{H_0} \tag{27}$$

Quantitatively it represents the original Hubble's law and qualitatively differs from the modern cosmic acceleration.

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Simple Algorithms Of The Sequential Growth Of The X-graph

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Abstract. Algorithms of sequential growth of x-graph are considered. This model is a particular case of a causal set approach to quantum gravity. The x-graph is a directed acyclic dyadic graph. The sets of vertices and edges of x-graph are particular cases of causal sets. The sequential growth of a graph is an addition of new vertices one by one. Five simple stochastic algorithms are considered. These algorithms satisfy the causality principle. Only a maximal or minimal vertex can be added. The probability to add a new maximal (minimal) vertex depend on the part of the x-graph that precedes (follows) this vertex.

Keywords: random graph, directed graph, causal set.
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1. INTRODUCTION

A particular case of a causal set is considered. A causal set approach to quantum gravity was introduced by G. 't Hooft [1] and J. Myrheim [2] in 1978 (see e.g. [3]). The model of a pregeometry is a directed acyclic dyadic graph. The directed graph means that all edges are directed. The dyadic graph means that each vertex has two incident incoming edges and two incident outgoing edges. This model was introduced by D. Finkelstein [4] in 1988. The acyclic graph means that there is not a directed loop. Hereinafter only such graph is considered and it is called an x-graph. The set of vertices and the set of edges of x-graph are causal sets.

The far goal of this model is to describe particles as self-organized repetitive structures of the x-graph. The simplest repetitive structure is a sequence of double edges (Figure 1a). Each inextendible antichain of edges includes 2 edges. An inextendible antichain (a slice) is defined as set of pairwise acausal edges. We can prove that in the x-graphs all slices of edges have the same cardinality [5, Theorem 5]. The second example is a repetitive structure that each antichain of edges includes 3 edges (Figure 1b). We can make a lot of such structures by hand. But the structures must emerge as a consequence of a dynamics.

The model of the universe is an infinite x-graph. But any observer can consider only finite x-graphs. In a graph theory, by definition, an edge is a relation of two vertexes. Consequently some vertexes of finite x-graph have less than four incident edges. These vertexes have free valences instead the absent edges. These free valences are called external edges as external lines in Feynman diagrams. They are figured as edges that are incident to only one vertex. External edges are not real edges. They are a property of vertices. But often it is useful to consider external edges as edges. There are incoming and outgoing external edges. We can prove that in the x-graphs the number of incoming external edges is equal to the number of outgoing external edges [5, Lemma 5].

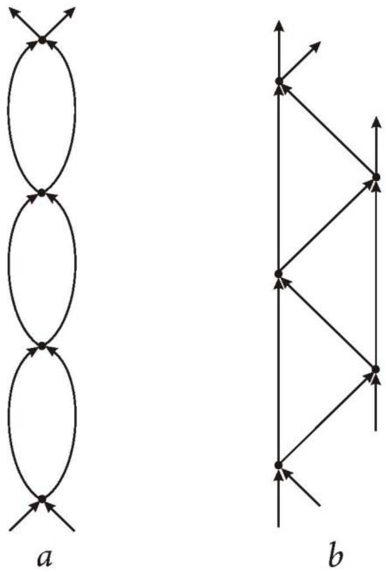

FIGURE 1. Simple repetitive structures.

Each x-graph is a model of a part of some process. The task is to predict the future stages of this process or to reconstruct the past stages. We can reconstruct the x-graph step by step. The minimal part is a vertex. We start from some given x-graph and add new vertices one by one. This procedure is proposed by author in 1998 [6, 7]. Similar procedure and the term `a classical sequential growth dynamics' are proposed by D. P. Rideout and R. D. Sorkin for other model of a causal set in 1999 [8].

We can add new vertices only to external edges. This procedure is called an elementary extension. There are four types of elementary extensions (Figure 2). In this and following figures the x-graph *G* is represented by a rectangle because it can have an arbitrary structure. The edges that take part in the elementary extension are figured by bold arrows. The new vertex is a maximal or minimal element of the causal set of vertices. There are two types of elementary extensions to the future. First type is an addition of a new maximal vertex to two outgoing external edges (Figure 2a). Second type is an addition of a new maximal vertex to one outgoing external edge (Figure 2b). Similarly, there are two types of elementary extensions to the past. Third type is an addition of a new minimal vertex to two incoming external edges (Figure 2c). Fourth type is an addition of a new minimal vertex to one incoming external edge (Figure 2d). We can prove that we can get every connected x-graph by a sequence of elementary extensions of these 4 types [5, Teorem 2].

By assumption, the dynamics of this model is a stochastic dynamics. We can only calculate probabilities of different variants of elementary extensions. In general case, these probabilities depend on the structure of an existed x-graph.

Consider simple combinatorial rules to calculate probabilities. All algorithms include 3 steps. The first step is the choice of the elementary extension to the future or to the past. We assume time symmetry. Then the probability of this choice is 1/2. The second step is the choice of the first external edge that takes part in the elementary extension. Denote the probability to choose the external edge number i by pi. The third step is the choice of the second external edge number j that takes part in the elementary extension. Denote the probability to choose the external edge number j by pij if we chose the external edge number i at the second step. These probabilities depend on the structure of the existed x-graph.

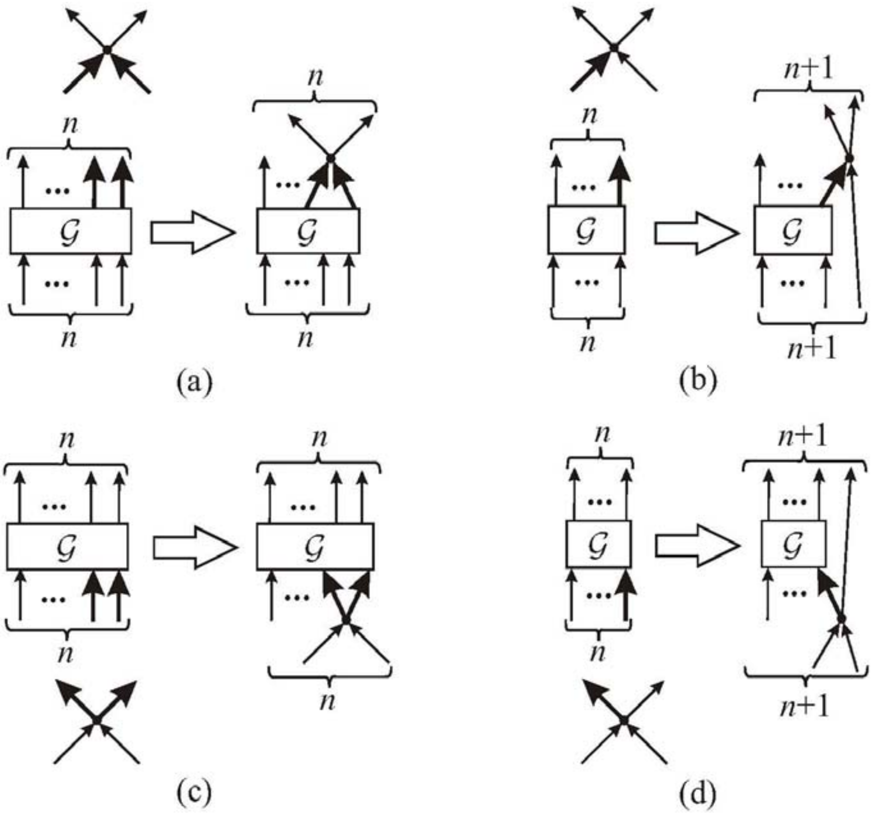

FIGURE 2. Elementary extensions: (a) the first type; (b) the second type; (c) the third type; (d) the fourth type.

In the considered model, we assume the causality principle in the following form. The probability to add a new maximal vertex can depend only on the subgraph that precedes this vertex [8]. Similarly, the probability to add a new minimal vertex can depend only on the subgraph that follows this vertex.

Consider simple variants of the sequential growth dynamics.

2. TWO SIMPLEST ALGORITHMS

There are two constants that describe the size of x-graph. These are the number N of vertices and the number n of incoming (or outgoing) external edges. The simple normalization of probabilities is proportional to N^{-1} or n^{-1}. If we chose n^{-1} and we assume that $p_{ij} = p_{ji}$, we get a unique algorithm [9]. We can introduce this algorithm in the following simple form [10] (Figure 3a). Let choose the elementary extension to the future (the variant to the past is the same). We choose the incoming external edge. We assume the equiprobable choice. The probability of this choice is $1/n$. Then we choose two directed paths from this incoming external edge. We must choose 1 edge in each vertex of the path. Consequently if this is equiprobable choice, the choice of this path has the probability 2^{-k}, where a directed path includes k vertexes. Each directed path ends in some outgoing external edge numbers i and j. A new vertex is added to these outgoing external edges. If these edges coincide, this is the addition of a new vertex to one outgoing external edge (Figure 2b). We get the right normalization of probabilities because the path ends in one and only one outgoing external edge in any case.

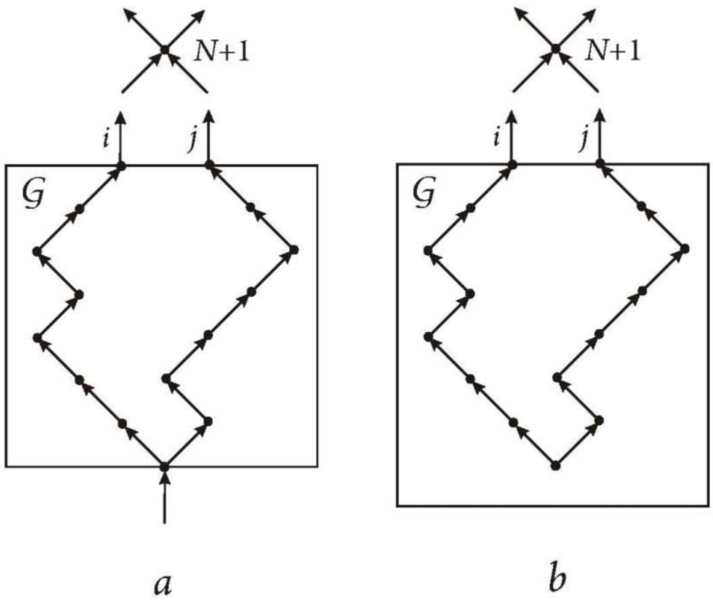

FIGURE 3. (a) The first algorithm. (b) The second algorithm.

If we chose N^{-1} as a normalization constant and we assume that $p_i p_{ij} = p_j p_{ji}$, we get a second unique algorithm [11]. We can introduce this algorithm in the following simple form (Figure 3b). The procedure is similar to the previous. But we choose the vertex instead the incoming external edge. We assume the equiprobable choice. The probability of this choice is $1/N$.

These are the simplest algorithms that satisfy the causality principle. But they cannot describe decay. Consider the decay of some process (Figure 4). A new separate part G_2 of the x-graph must emerge during the sequential growth. In the considered algorithms, if the first directed path ends in G_2, the second directed path must end in G_2 too. Such addition of new vertices to G_2 must repeat many times.

But this has a small probability in the big x-graph. Often the second directed path ends in the other part G_1 of the x-graph.

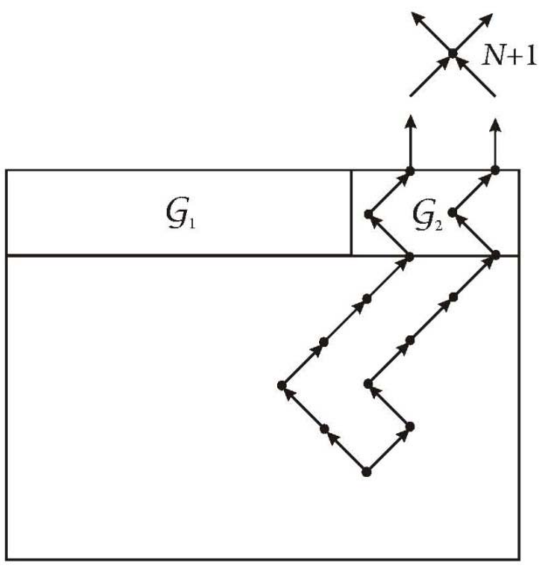

FIGURE 4. The decay.

3. THE THIRD ALGORITHM

Consider a modification of the algorithm to avoid the problem of decay. The first and second steps are the same as in the first algorithm. We choose the elementary extension to the future or to the past with probability 1/2 and the first external edge number i that takes part in this elementary extension with probability $1/n$. Let choose the elementary extension to the future (the variant to the past is the same). Then we choose an opposite directed paths from external edge number i (Figure 5). In each vertex, we choose a continuation of the opposite directed path or a turn with probability 1/2. If we choose the continuation, we choose one edge with probability 1/2. If we choose an internal edge, we go to the next vertex and repeat this process. If we choose an incoming external edge, we must turn. After the turn in some vertex number V we choose the directed path to some outgoing external edge number j as in the previous algorithm.

In this algorithm, the probability to choose the vertex of the turn exponentially decreases depending on the distance between this vertex and the initial outgoing external edge. We have the high probability to add a new vertex to small separated parts of the x-graph.

4. ALGORITHMS WITH A FREE PARAMETER

The considered algorithms have not free parameters. The probability of the addition of a new vertex to one external edge is a function of the structure of the x-graph. But this elementary extension describes the interaction of the considered system with environment. This interaction must depend on the environment.

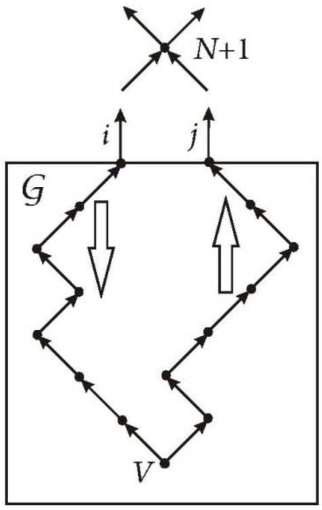

FIGURE 5. The third algorithm.

Consider a simple case. Let choose the elementary extension to the future (the variant to the past is the same). The probability of the addition of a new vertex to one outgoing external edge is equal to p. We assume the equiprobable choice. Then the probability to add a new vertex to any outgoing external edge is equal to p/n. The probability of the addition of a new vertex to two outgoing external edges is equal to q=1-p. We can consider some algorithm to choose the addition of a new vertex to two outgoing external edges that is normalized by 1 and renormalized it by q.

We cannot use the previous algorithms because if we remove the paths that return to the initial edge, we violate the normalization. Consider a modification of the last algorithm. If we return to the initial outgoing external edge, we must choose the second outgoing edge that incident to the same vertex (Figure 6). If this is outgoing external edge, we choose this edge (Figure 6a). If this is outgoing internal edge, we continue the directed path (Figure 6b).

This algorithm looks unnatural. Consider the fifth algorithm with edge disjoint paths (Figure 7). The path from the initial outgoing external edge number i to the vertex number V of the turn cannot have a common edge with the path from the vertex number V to the second outgoing external edge number j.

We get the following algorithm. We start from the first outgoing external edge number i that incident on the vertex number A. Then we choose an opposite directed path from the edge number i. In each vertex (A, B…), we choose the continuation of the opposite directed path or the turn with probability 1/2. If we choose the continuation, we choose one edge with probability 1/2. If we choose an internal edge, we go to the next vertex and repeat this process. If we choose some incoming external edge number α, we must turn (Figure 7a). After the turn in the incoming external edge number α or in the vertex number V (Figure 7b) we choose the directed path to some outgoing external edge number j. In each vertex we choose one next edge with probability ½ (the vertex number C in the figure). If we came to the vertex, that is included in the opposite directed path (the vertex number D in the figure), we must choose the edge that is not included in the opposite directed path. The path ends in some outgoing external edge number j.

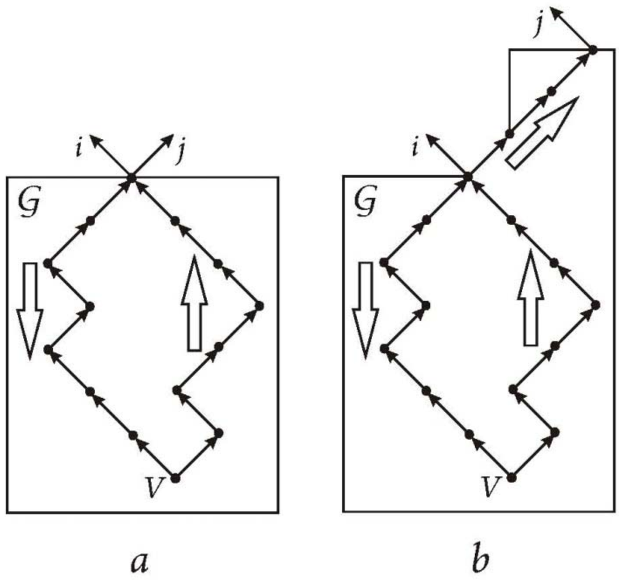

FIGURE 6. The algorithm with the free parameter.

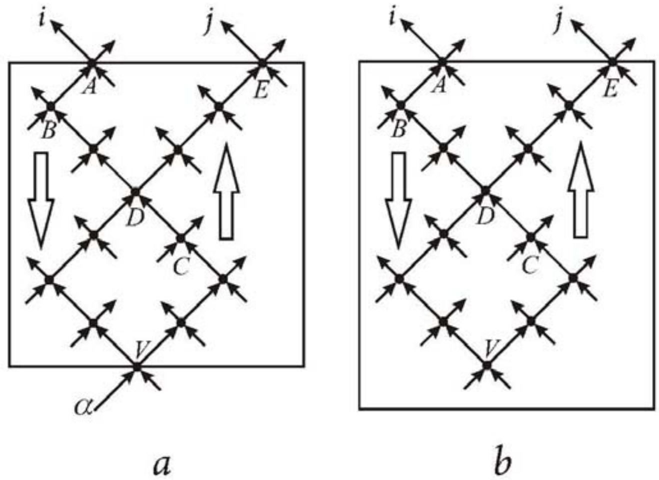

FIGURE 7. The algorithm with edge disjoint paths.

The free parameter p describes the intensity of the interaction of the considered system with environment. In two extreme cases, p=1 or p=0. This is a maximal and minimal interaction with environment. If we start from a single vertex, we get a tree in the first case and a sequence of double edges (Figure 1a) in the second case. It is possible to have a phase transition in some value of p.

Consider the algebraic approach to the calculation of probabilities of edge disjoint paths [12]. Number all internal and external edges of the x-graph. Denote the numbers of edges by lowercase Latin indices. Assign quantity e_a to each edge number a. Number all vertices of the x-graph. Denote the numbers of vertices by capital Latin indices. Assign quantity v_A to each vertex number A. By $Q_{m(ij)}$ denote the edge disjoint path from the outgoing external edge number i to the outgoing external edge number j. The lowercase Latin index m is the number of the path.

We have $Q_{1(ij)}= e_i\ ½v_A\ e_a\ ½v_B\ e_b \dots ½v_D \dots ½v_V\ e_\alpha\ v_V \dots v_C\ e_c\ v_D\ e_d \dots v_E\ e_j$ for the path in Figure 7a. We have $Q_{2(ij)} = e_i\ ½v_A\ e_a\ ½v_B\ e_b \dots ½v_D \dots ½v_V \dots v_C\ e_c\ v_D\ e_d \dots v_E\ e_j$ for the path in Figure 7b. We have the factor ½ for each vertex before the turn. This factor describes the probability to choose the turn or not. Each quantity v_A describes the choice of one of two edges in the vertex. But if the vertex is included twice in the path, we have the choice of edge only in the first time. We have $v_A\ v_A = v_A$. Each quantity v_A is idempotent. If $Q_{m(ij)}$ includes some edge twice, this path is not allowed. We have $Q_{m(ij)}$=0. Then $e_a\ e_a = 0$. Each quantity e_a is nilpotent. Consider two operators $\boldsymbol{E}$ and $\boldsymbol{V}$. By definition, put $\boldsymbol{E}(e_a)$=1 and $\boldsymbol{V}(v_A)$=½. By $p(Q_{m(ij)})$ denote the probability to choose the path $Q_{m(ij)}$. We get $p(Q_{m(ij)})=\boldsymbol{EV}(Q_{m(ij)})$. The probability p_{ij} is equal to the sum over all $p(Q_{m(ij)})$ of allowable paths from the edge number i to the edge number j.

5. DISCUSSION

Many cases of deterministic algorithms are investigated by S. Wolfram for cellular automata and other simple models [13]. Some of deterministic algorithms for causal sets are investigated by T. Bolognesi in [14]. In some cases we can get self-organization, highly complex patterns, repetitive or nested structures by using very simple algorithms.

In this paper the simple stochastic algorithms are considered. These algorithms are handy for a numerical simulation. If we start from a single vertex, we can calculate all probabilities for the first four algorithms by iterative procedures. We calculate all probabilities at the step number N+1 by using the probabilities at the step number N and some additional quantities. The number of calculations at each step is proportional to n^2. These procedures are described in [9] and [11] for the first and second algorithms respectively. It is not difficult to get such procedures for the third and fourth algorithms. But this is not the case for the fifth algorithm. In this algorithm, there is an interaction of directed path and opposite directed path. We must calculate all paths to get all probabilities. This is impossible for big x-graphs. But we do not need all probabilities. We can choose the variant of the elementary extension by a random walk at the x-graph. The calculation for the choice of the first external edge is proportional to n. The probability of the random walk exponentially decreases depending on the length of this walk. In the majority of cases, we need small number of calculations for the choice of the second external edge that approximately does not depend on the size of the x-graph for the big x-graph.

There are some results of a numerical simulation for the first algorithm in [15]. The numerical simulation of the next algorithms is a work in progress.

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The Behavior Of The Strange Quark Matter For FRW Model With Time Varying Constants

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Abstract; In this paper we study the behavior of the strange quark matter for FRW model with time varying constants. We find the solutions through the Lie method and show that $\frac{G}{c^2}$ = constant, is correct from the mathematical point of view. We also discuss three cases and observe that the constants G, c and Λ are decreasing functions on time, while the scale factor R is increasing function on time and ρ is a decreasing function on time.

Keywords: VSL theory, strange quark matter, bag constant.
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1. INTRODUCTION

Since the pioneering work of Dirac [1], a theory with a time variable gravitational coupling constant G and cosmological term Λ , have been intensively investigated in the physical literature. Belinchon [2] have studied the behavior of the constants G, c and Λ , with the framework of a flat Friedman-Robertson-Walker (FRW) cosmological model, where the effects of a c-variable into the curvature tensor have been studied.

The cosmological effects of introducing a time variation of speed of light, c, into the gravitational field equations, have revealed a number of tantalizing possibilities. If the speed of light falls sufficiently rapidly over an interval of time then it is possible to solve the standard horizon and flatness problems in a way that differs from inflationary universe.

There are some criticisms about the varying c models. For instance, the speed of light is not dimensionless quantity; hence, going to a new frame, one may cancel its probable variations. Or, if c and, consequently, the coupling constant of the Einstein's equations would vary, then observers in different frames would see the evolution of the Universe governed by different rules. However, these arguments are also applicable to any varying-constant model, in which some physical constants are made to vary. Variable Speed of Light (VSL) models proposed by Moffat [3] and Alrecht and Magueijo [4] in which light was travelling faster in the early periods of the existence of the universe, might solve the same problems as inflation. Einstein's field equations for FRW space-time in the VSL theory have been solved by Barrow [5], who also obtained the rate of variation of the speed of light require to solve the flatness and the cosmological constants problems (see Magueijo [6] for a review of these theories). Some authors [7] – [8] have proposed a new generalization of general relativity which also allows arbitrary changes in the speed of light, *c*, and the gravitational constant, *G*, but in such a way that the variations in the speed of light introduce correction to the curvature tensor in the Einstein equations in the cosmological frame.

The dimensional analysis has the structure of a Lie group [9]. The Lie group have been performed by [10] – [11], where they study the Friedman equations in order to find the correct equation of state, following pioneer work of [12]. We would like to emphasize that the Lie group method shows us that one of the assumptions $\frac{G}{c^2} =$ constant, is correct from the mathematical point of view.

The possibility of the existence of quark matter dates back to early 1970's. Bodmer [13], Itoh [14], and Witten [15] proposed two ways of formation of quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction quark bag models suppose that breaking of physical vacuum takes place inside hadrons. As a result vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of the quarks, thus stabilizing the system. If the hypothesis of the quark matter is true, then some of the neutrons stars could actually be strange stars, built entirely of strange matter [16] – [17].

Typically, strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume [18]. In this model, quarks are thought as degenerate Fermi gases, which exist only in region of space endowed with a vacuum energy density B_c (called as bag constant).

Also in the framework of this model the quark matter is composed of mass less *u, d* quarks, massive s quarks and electrons. In the simplified version of this model, on which our study is based, quarks are mass less and non-interacting. Then we have quark pressure $p_q = \frac{\rho_q}{3}$, where ρ_q is the quark energy density, the total energy density $\rho = \rho_q + B_c$ and total pressure $p = p_q - B_c$. One therefore get equation of state (EOS) for strange quark matter [19],

$$p = \frac{1}{3}(\rho - 4B_c). \tag{1}$$

In this work we consider flat FRW cosmological model, filled with quark-gluon plasma, as a perfect fluid and also assume that G, c and Λ are time dependent, in the context of general theory of relativity. Once we have obtained the field equations, we rewrite them in order to obtain second order differential equation, in order to apply the standard Lie procedure, in the presence of strange quark matter. The paper is organized as follows.

In section 2, we have obtained the Einstein field equations for strange quark matter in FRW space time by assuming $div(T_{ij}) = 0$. In section 3, we obtained the possible solutions to the field equations using Lie group method and discussed the behavior of constants G, c, Λ, ρ by assuming three different cases. Section 4 ends with a brief conclusion.

2. THE MODEL AND THE FIELD EQUATIONS

We consider the FRW metric of the form,

$$ds^2 = -c^2(t)dt^2 + d\Omega^2 \ , \tag{2}$$

with

$$d\Omega^2 = R(t)^2\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right) \ . \tag{3}$$

The field equations with G, c and Λ can be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G(t)}{c^4(t)}T_{ij} + \Lambda(t)\, g_{ij} \ , \tag{4}$$

where the energy momentum tensor is

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \ , \tag{5}$$

and the 4-velocity μ^i is defined as $\mu^i = (c^{-1}, 0, 0, 0)$ such that $\mu^i \mu_i = -1$.

The equation of state of the strange quark matter can be written as $p = \frac{1}{3}(\rho - 4B_c)$.

We consider the flat model i.e. k=0 then cosmological equations can be written are as follows:

$$2\dot{H} + 3H^2 = -\frac{8\pi G(t)}{c(t)^2}p + c(t)^2\Lambda(t), \tag{6}$$

$$3H^2 = \frac{8\pi G(t)}{c(t)^2}\rho + c(t)^2\Lambda(t), \tag{7}$$

where dot(.) denotes derivative with respect to t and $H = \frac{\dot{R}}{R}$, is the Hubble function.

Applying the covariance divergence to the R. H. S. of Eq. 3 ,we get,

$$\mathrm{div}\left(\frac{8\pi \mathrm{G(t)}}{\mathrm{c}^4\mathrm{(t)}}T_{ij} + \Lambda(\mathrm{t})\mathrm{g}_{\mathrm{ij}}\right) = 0, \tag{8}$$

which simplified as,

$$\mathrm{T}_{\mathrm{i;j}}^{\mathrm{j}} = \frac{c^4}{G}\left(\frac{-\delta_{\mathrm{i}}^{\mathrm{j}}\Lambda_{,j}}{8\pi} + \frac{4GT_i^{\ j}c_{,j}}{c^5} - \frac{G_{,j}T_i^{\ j}}{c^4}\right). \tag{9}$$

With the help of equation of state for strange quark matter we get

$$\dot{\rho} + 4(\rho - B_c)H = \frac{-\dot{\Lambda}c^4}{8\pi G} - \rho\frac{\dot{G}}{G} + 4\rho\frac{\dot{c}}{c} \tag{10}$$

By the principle of conservation for its energy-momentum tensor, we assume that

$$\mathrm{div}(T_{ij}) = 0,$$

then Eq. 9 is reduced to

$$\dot{\rho} + 4(\rho - B_c)H = 0, \tag{11}$$

$$\frac{-\dot{\Lambda}c^4}{8\pi G} - \rho\frac{\dot{G}}{G} + 4\rho\frac{\dot{c}}{c} = 0 \tag{12}$$

Hence the above field equations are

$$2\dot{\mathrm{H}} + 3\mathrm{H}^2 = -\frac{8\pi \mathrm{G(t)}(\rho - 4\mathrm{B_c})}{3\mathrm{c(t)}^2} + \mathrm{c(t)}^2\Lambda(\mathrm{t}), \tag{13}$$

$$3\mathrm{H}^2 = \frac{8\pi \mathrm{G(t)}}{\mathrm{c(t)}^2}\rho + \mathrm{c(t)}^2\Lambda(\mathrm{t}), \tag{14}$$

$$\dot{\rho} + 4(\rho - B_c)H = 0, \tag{15}$$

$$\frac{-\dot{\Lambda}c^4}{8\pi G} - \rho\frac{\dot{G}}{G} + 4\rho\frac{\dot{c}}{c} = 0. \tag{16}$$

3. LIE METHOD

In this section we obtain the solutions of the field equations by using the Lie method. In order to use the Lie method, we rewrite the field equations as follows. From Eqs. 12 - Eq. 15, we get

$$\frac{2\ddot{\mathrm{R}}}{R} - \frac{2\dot{R}^2}{R^2} = \frac{-32\pi G}{3c^2}(\rho - B_c). \tag{17}$$

and therefore,

$$2\dot{H} = \frac{-32\pi G}{3c^2}(\rho - B_c). \tag{18}$$

From Eq. 14, we get

$$H = \frac{-\dot{\rho}}{4(\rho - B_c)}, \tag{19}$$

therefore,

$$\left(\frac{\dot{\rho}}{\rho - B_c}\right)^{\cdot} = \frac{64\pi G}{3c^2}(\rho - B_c). \tag{20}$$

Taking $\frac{64\pi}{3} = A$ and then expanding, we get

$$\ddot{\rho} = \left(\frac{\dot{\rho}^2}{\rho - B_c}\right) + \frac{AG}{c^2}(\rho - B_c)^2, \tag{21}$$

which can also be written as,

$$(\rho - B_c)^{\cdot\cdot} = \frac{(\rho - B_c)^{\cdot 2}}{(\rho - B_c)} + \frac{AG}{c^2}(\rho - B_c)^2. \tag{22}$$

We denote for further calculation,

$$(\rho - 4B_c) = \rho_\tau. \tag{23}$$

Then Eq. 21 can be written as

$$\ddot{\rho}_\tau^{\ 2} = \frac{\dot{\rho}_\tau^{\ 2}}{\rho_\tau} + \frac{AG}{c^2}\rho_\tau^{\ 2}. \tag{24}$$

Now, we apply the standard Lie procedure to this equation. A vector field X

$$X = \xi(t, \rho_\tau)\partial t + \eta(t, \rho_\tau)\partial \rho, \tag{25}$$

is a symmetry of Eq. 23 iff

$$\begin{gathered} -\xi f_t - \eta f_{\rho_\tau} + \eta_{tt} + (2\eta_{t\rho_\tau} - \xi_{tt})\dot{\rho} + (\eta_{\rho_\tau\rho_\tau} - 2\xi_{t\rho_\tau})\dot{\rho}^2 - \xi_{\rho_\tau\rho_\tau}\dot{\rho}_\tau^{\ 3} \\ + (\eta_{\rho_\tau} - 2\xi_t - 3\dot{\rho}_\tau\xi_{\rho_\tau})f - [\eta_t + (\eta_{\rho_\tau} - \xi_t)\,\rho_\tau - \dot{\rho}_\tau^{\ 2}\xi_{\rho_\tau}]f_{\dot{\rho}_\tau} = 0 \end{gathered} \tag{26}$$

By expanding and separating Eq. 25 with respect to powers of $\dot{\rho}_\tau$, we obtain the system of differential equations

$$\xi_{\rho_\tau\rho_\tau} + \rho_\tau^{\ -1}\xi_{\rho_\tau} = 0, \tag{27}$$

$$\eta_{\rho_\tau\rho_\tau} - 2\xi_{t\rho_\tau} + \rho_\tau^{\ -2}\eta - \rho_\tau^{\ -1}\eta_{\rho_\tau} = 0, \tag{28}$$

$$2\eta_{t\rho_\tau} - \xi_{tt} - 3A\frac{G}{c^2}\rho_\tau^{\ 2}\xi_{\rho_\tau} - 2\rho_\tau^{\ -1}\eta_t = 0, \tag{29}$$

$$\eta_{tt} - A\left(\frac{\dot{G}}{c^2} - 2G\frac{\dot{c}}{c^3}\right)\rho_\tau^{\ 2}\xi - 2\eta A\frac{G}{c^2}\rho_\tau + (\eta_{\rho_\tau} - 2\xi_t)A_{\rho_\tau}\frac{G}{c^2}\rho_\tau^{\ 2} = 0, \tag{30}$$

Solving Eq. 26 - Eq. 29, we find that

$$\xi(t, \rho_T) = -2et + a, \qquad \eta(t, \rho_\tau) = (bt + f)\rho_\tau, \tag{31}$$

subject to the constrain,

$$\frac{\dot{G}}{G} = \frac{2\dot{c}}{c} + \frac{bt + f - 4e}{2et - a}, \tag{32}$$

with a, b, f and e are constants. In order to solve Eq. 31, we consider the following three cases.

3.1. Case I : $b = 0$ and $f - 4e = 0$

In this case, Eq.(31) reduces to

$$\frac{\dot{G}}{G} = \frac{2\dot{c}}{c} \quad \Rightarrow \quad \frac{G}{c^2} = B = constant, \tag{33}$$

which means that constants G and c vary but in such a way that the relation $\frac{G}{c^2}$ remains constant.

In order to obtain the complete solutions of the field equations, consider the above Eq. 32 as a hypothesis. This case shows us that such hypothesis is correct from the mathematical sense. The knowledge of one symmetry X might suggest the form of a particular solution as an invariant of the operator X i.e. the solution of

$$\frac{dt}{\xi(t, \rho_\tau)} = \frac{d\rho_\tau}{\eta(t, \rho_\tau)}. \tag{34}$$

This particular solution is known as an invariant solution. Therefore, the quark energy density ρ is obtained as

$$\frac{dt}{(-2et + a)} = \frac{d\rho_t}{4e\rho_\tau} \quad \Rightarrow \quad \rho_\tau = \rho_0 (2et - a)^{-2}. \tag{35}$$

With the help of Eq. 22 we get

$$\rho = \rho_0 (2\mathrm{et} - \mathrm{a})^{-2}, \tag{36}$$

for simplicity, we adopt

$$\rho = \rho_0 t^{-2} + B_c. \tag{37}$$

From Eq. 12, we can obtain ρ , and hence R (the scale factor) as :

$$\rho = B_c + c_1 R^{-4} \quad \Rightarrow \quad R = A_0 t^{\frac{1}{2}}, \tag{38}$$

where c_1 is the constant of integration and $A_0 = \left(\frac{c_1}{\rho_0}\right)^{\frac{1}{4}}$. In this way we find H and from Eq. 13, we obtain the behavior of Λ as

$$c^2 \Lambda = 3H^2 - \frac{8\pi G}{c^2}\rho, \tag{39}$$

and therefore,

$$\Lambda = \frac{l}{c^2 t^2} - \frac{8\pi B B_c}{c^2}, \tag{40}$$

where, $l = \frac{3}{4} - 8\pi B\rho_0$. If we replace all these results into Eq. 15, then we obtain the exact behavior for *c*, i.e.,

$$\frac{\dot{c}}{c} = \frac{-l}{4\pi BtF_1(t)F_2(t)}, \tag{41}$$

where, $F_1(t) = \rho_0 + B_c t^2$, $F_2(t) = \frac{l}{4\pi BF_1(t)} - \frac{2B_c t^2}{F_1(t)} + 2$, and thus,

$$c = c_0 t^{-\alpha}, \tag{42}$$

where $\alpha = \frac{\lambda}{\lambda + \rho_0}$, $\lambda = \frac{l}{8\pi B}$.

Thus in this case we have found that

$$G = G_0 t^{-2\alpha}, \quad \rho = \rho_0 t^{-2} + B_c, \quad c = c_0 t^{-\alpha},$$
$$R = A_0 t^{\frac{1}{2}}, \quad \Lambda = \Lambda_0 t^{-2(1-\alpha)} - \frac{8\pi BB_c}{c_0^2} t^{2\alpha}, \tag{43}$$

3.2. Case II : b = a = 0

In this case, the Eq. 31 reduces to

$$\frac{\dot{G}}{G} = \frac{2\dot{c}}{c} + \frac{f - 4e}{2et} \quad \Rightarrow \quad \frac{G}{c^2} = Bt^{k_0}, \tag{44}$$

where $k_0 = \delta - 2$ and $\delta = \frac{f}{2e}$.

Similarly by using the above procedure, we find that

$$\frac{dt}{\xi(t, \rho_\tau)} = \frac{d\rho_\tau}{\eta(t, \rho_\tau)} \quad \Rightarrow \quad \rho_\tau = \rho_0 t^{-\delta}. \tag{45}$$

Again by using Eq. 22 we get,

$$\rho = \rho_0 t^{-\delta} + B_c. \tag{46}$$

We must impose the condition $\delta \in \Re^+$. This solution has a physical sense since the quark energy density ρ is decreasing function of time. It is also observed that if $f = 4e$ then we obtain same solution that we obtained in case I. The scale factor is found to be

$$R = A_0 t^{\frac{\delta}{4}}, \tag{47}$$

where A_0 is constant. Hence the Hubble parameter is

$$H = \frac{\delta}{4t}, \quad i.e. \quad H \propto \frac{1}{t}. \tag{48}$$

To obtain the behavior of the constants G, c and Λ, we follow the same steps as in case I, i.e. from Eq. 13, we get

$$c^2\Lambda = 3H^2 - \frac{8\pi G}{c^2}\rho, \quad (49)$$

and therefore,

$$\Lambda = \frac{l}{c^2 t^2} - \frac{8\pi BB_c t^{k_0}}{c^2}, \quad (50)$$

where $l = \frac{3\delta^2}{16} - 8\pi B\rho_0$ *i.e.* $l \in \Re^+$.

Therefore,

$$\dot{\Lambda} = -2l\left(\frac{1}{t} + \frac{\dot{c}}{c}\right) - \frac{8\pi BB_c t^{k_0-1}}{c^2}\left(k_0 - \frac{2t\dot{c}}{c}\right). \quad (51)$$

If we substitute all these results into the Eq. 15 we get

$$\frac{\dot{c}}{c} = \frac{F_4(t)}{tF_3(t)}, \quad (52)$$

where, $F_3(t) = \frac{l}{4\pi B} - 2B_c t^{\delta} + 2F_1(t)$, $F_4(t) = \frac{-l}{4\pi B} - kB_c t^{\delta} + k_0 F_1(t)$

and $\frac{F_4(t)}{F_3(t)} \in \Re^-$ and thus,

$$c = c_0 t^{-\alpha}, \quad (53)$$

with $\alpha = \left[\frac{\lambda - \frac{k_0\rho_0}{B_c}}{\lambda + \frac{2\rho_0}{B_c}}\right]$ and $\lambda = \frac{l}{4\pi BB_c}$ such that $\alpha \in [0,1)$.

In this way we can find the rest of the quantities

$$G = G_0 t^{-2\alpha + k_0}, \quad \rho = \rho_0 t^{-\delta} + B_c, \quad c = c_0 t^{-\alpha},$$

$$R = A_0 t^{\frac{\delta}{4}}, \quad \Lambda = \Lambda_0 t^{-2(1-\alpha)} - \frac{8\pi BB_c t^{(k_0+2\alpha)}}{c_0^2}. \quad (54)$$

We notice that this solution is very similar to the case I, but in this case all the parameters are perturbed by δ and more important is the result $\frac{G}{c^2} = Bt^{k_0}$.

3.3. Case III : b = e = 0

Following the same procedure as above, we find in this case that Eq. 30

$$\Rightarrow \xi(t, \rho_T) = a, \quad \eta(t, \rho_\tau) = f\rho_\tau$$

and therefore,

$$\frac{\dot{G}}{G} = \frac{2\dot{c}}{c} - \frac{f}{a}. \quad (55)$$

After integration we get

$$\frac{G}{c^2} = K \exp(-\alpha t), \quad (56)$$

where $\alpha = \frac{f}{a}$ and K is constant of integration.

$$\frac{dt}{\xi(t,\rho_\tau)} = \frac{d\rho_\tau}{\eta(t,\rho_\tau)} \quad \Rightarrow \quad \frac{dt}{\alpha} = \frac{d\rho_\tau}{f\rho_\tau} \quad \Rightarrow \quad \rho_\tau = \rho_0 e^{\alpha t}. \tag{57}$$

With the help of Eq. 22 we get,

$$\rho = \rho_0 e^{\alpha t} + B_c. \tag{58}$$

This equation only has sense if $\alpha \in \Re^-$. The scale factor R satisfies the relationship

$$\rho = B_c + c_1 R^{-4} \quad \Rightarrow \quad R = A_0 \exp\left(\frac{-\alpha t}{4}\right). \tag{59}$$

That is to say, it is a growing function without singularity. In this way, we find that

$$H = \frac{-\alpha}{4} = constant. \tag{60}$$

To obtain the behavior of the constants G, c and Λ, we follow the steps as in case I, i.e. from Eq. 13, we get,

$$c^2\Lambda = 3H^2 - \frac{8\pi G}{c^2}\rho, \tag{61}$$

and therefore,

$$c^2\Lambda = l - 8\pi K B_c e^{-\alpha t}, \tag{62}$$

where, $l = \frac{3\alpha^2}{16} - 8\pi K\rho_0$. Therefore

$$\Lambda = \frac{1}{c^2}\left(l - 8\pi K B_c e^{-\alpha t}\right), \tag{63}$$

and hence

$$\dot{\Lambda} = \frac{-2}{c^2}\left[\frac{l\dot{c}}{c} - 4\pi K B_c e^{-\alpha t}\left(\alpha + \frac{2\dot{c}}{c}\right)\right]. \tag{64}$$

If we replace all these results into Eq. 15 then we obtain,

$$\frac{\dot{c}}{c} = \frac{F_7(t)}{F_6(t)}, \tag{65}$$

where, $F_6(t) = \left[\frac{l}{4\pi K \exp(-\alpha t) F_5(t)} - \frac{2B_c}{F_5(t)} + 2\right]$ and $F_7(t) = \left[\frac{\alpha B_c}{F_5(t)} - \alpha\right]$,

and hence,

$$c = K_1 \exp(c_0 t), \tag{66}$$

where $c_0 = -\left(\frac{4\pi\alpha K\rho_0}{l + 8\pi K\rho_0}\right)$ and K_1 is constant of integration with $c_0 \in \Re^-$. *i.e.* c is a decreasing function on time t. In this way we can find the rest of the quantities. Hence we have

$$G = G_0 \exp(-\alpha + 2c_0)t, \quad \rho = \rho_0 e^{\alpha t} + B_c, \quad c = K_1 e^{c_0 t}, \ R = A_0 \exp\left(\frac{-\alpha t}{4}\right),$$

$$\Lambda = \Lambda_0 \exp(-2c_0 t) - \frac{8\pi K B_c}{K_1^2}\exp(-\alpha - 2c_0)t. \tag{67}$$

4. CONCLUSION

The constant c was first introduced as the speed of light. However, with the development of physics, it came to be understood as playing a more fundamental role, its significance being not directly that of a usual velocity and one might thus think of c as being a fundamental constant of the universe. In this paper we have studied the behavior of time-varying constants G, c and Λ, for flat FRW space time in the presence of strange quark matter. To obtain the solution using the Lie group tactic we have imposed the condition, $div(T_{ij}) = 0$ and considered three different cases.

In **case I** when $b=0$ and $f-4e=0$, we get and relationship, $\frac{G}{c^2}$ = constant = B, remaining constant for all values of t, i.e. G and c vary but in such a way that $\frac{G}{c^2}$ remains constant. It is also observed that G, c are decreasing function and the cosmological term Λ is also a decreasing function of t for negative value of α and scale factor R is increasing and quark energy density ρ is a decreasing function of t. Hence, the solutions are physically relevant.

In **case II**, it is also observed that R is growing and quark density ρ is decreasing function on t, while G, c are decreasing function on t and the cosmological term Λ is also a decreasing function of t when k_0 and α are negative and it is also observed that Hubble parameter $H \propto \frac{1}{t}$. In this case we also note that $\alpha < 1$, because when $\alpha = 1 \Leftrightarrow \delta = 0$ is forbidden and $\alpha = 0$ brings us to the limiting case of G, Λ variable cosmologies.

In **case III**, we get inflationary solution (Yilmaz and Yavuz [20]). This is plausible, because the phase transition of quark-gluon plasma occurs in the early universe. The scale factor R is exponentially increasing and quark density ρ is exponentially decreasing function on t. In all the above three case it is also observed that when $\rho_0 = 0$ we get $\rho = B_{c,}$, the relation obtained earlier by Yilmaz and Yavuz [20] for flat FRW model. We have discussed three cases playing with constants a, b, e, f. More solutions can be obtained but we are interested only in the solutions with physical sense.

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On Pseudo-Superluminal Motion

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Abstract: Modern physics confirms the impossibility of Superluminal Motion through the considerations of Special Relativity. In General Relativity we may apply this constraint rigorously only to the Local Inertial Frames where Einstein's Field Equations are linear. This article, incidentally seeks to investigate the possibility of Pseudo-Superluminal motion in the non–local context without violating Special Relativity

Keywords: General Relativity, Local Inertial Frames, Manifold, Tangent Plane
PAC: 04.20.-q

INTRODUCTION

Finite speed of signal transmission is one of the greatest discoveries that have revolutionized modern physics. Special Relativity[1] through its second postulate claims that the speed of light is independent of its source. Though very much counter-intuitive if viewed through the "classical ideas" it turns out to be an amazing fact. In combination with the first postulate of relativity it leads to the novel aspect of space and time getting mixed up into a composite fabric. One of the fundamental outcomes of all this is the finite speed of signal transmission.

Incidentally all this refers to what we know as Flat Spacetime or Minkowski Space[2]. General Relativity is heavily based on the concept of the Local Inertial frames(LIF) which break up curved space into a set of small inertial territories. Curved Spacetime is governed by Einstein's Field equations which are non-linear in nature. But the Local Inertial frames offer us the advantage of Special Relativity---the Field Equations become linear. Calculations become simpler and comfortable in Flat Spacetime which exists here only in the local context, of course.

NON-LOCAL CONSIDERATIONS

Now we consider the observation of an event at a point Q from a point P such that they have a finite separation between them, so that both my not be located in the same Inertial local frame. But each point carries its own LIF with it. In our "thought experiment "we have two observers one at P and the other at Q. A light ray flashes across an infinitesimally small, spatial interval at Q. It is observed from both the points P and Q. The spatial interval noted by both is the same. But the time recorded for the passage is different for the observers since their clocks run at different rates, the metric coefficients pertaining to time , generally speaking , are different for the two points.

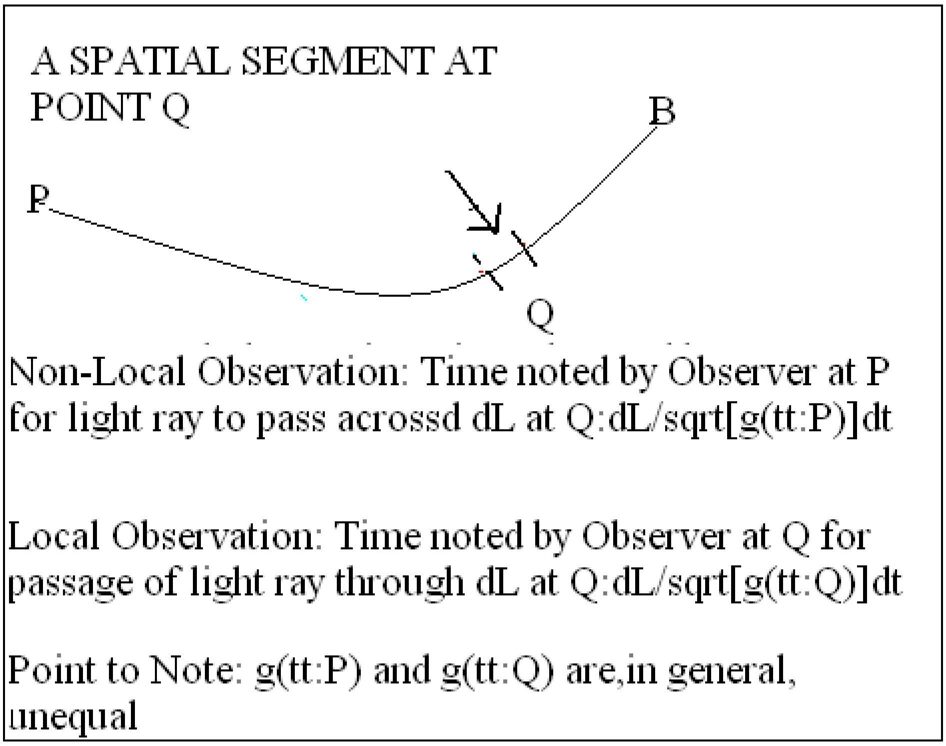

FIGURE 1. Local and Non-Local observations Metric[3]:

$$ds^2 = g_{tt}dt^2 - g_{zz}dx^2 - g_{yy}dy^2 - g_{zz}dz^2 \qquad (1)$$

Spatial interval at Q:

$$dL = \sqrt{g_{xx}(Q)dx^2 + g_{yy}(Q)dy^2 + g_{zz}(Q)dz^2}$$

Both the observers record the same value for the above.
Non-Local time interval observed from P:

$$dT_P = \sqrt{g_{tt}(P)}dt \qquad (2)$$

Local time interval observed from Q:

$$dT_Q = \sqrt{g_{tt}(Q)}dt \qquad (3)$$

Non-Local Observation: Speed of light at Q as observed from P: $c_P = \frac{dL}{\sqrt{g_{tt}(P)}dt}$

Local Observation: Speed of light at Q as observed from Q: $c_Q = \frac{dL}{\sqrt{g_{tt}(Q)}dt}$

But the speed of light as observed from Q is the local speed of light that is, $c_Q = c$,where c is the standard value for speed of light in vacuum as we know it.
Therefore,

$$\frac{c_P}{c_Q} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}$$

or

$$c_P = c_Q \times \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}.$$

Non-Local speed of ligh, c_P, is given by:

$$c_P = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times c \qquad (4)$$

[$c_Q = c$:Observation being Local]

Therefore the speed of light for non-local observation may be greater than, equal to, or less than the speed of light "c" as we know it, the standard value, depending on the value of the ratio $g_{tt(Q)} : g_{tt(P)}$

Now let's consider a *particle* moving across an infinitesimally small spatial interval at Q (instead of a light ray).

Spatial separation: $dL = \sqrt{g_{xx}(Q)dx^2 + g_{yy}(Q)dy^2 + g_{zz}(Q)dz^2}$

Both observers record the same value for it.

Time interval observed from P: $dT_P = \sqrt{g_{tt}(P)}dt$.

Time interval observed from Q: $dT_Q = \sqrt{g_{tt}(Q)}dt$

Non-Local Observation: Speed of particle at Q as observed from P:

$$v_{(P:particle)} = \frac{dL}{\sqrt{g_{tt}(P)}dt}$$

Local Observation: Speed of particle at Q as observed from Q:

$$v_{(Q:particle)} = \frac{dL}{\sqrt{g_{tt}(Q)}dt}$$

Therefore,

$$\frac{v_{(P:particle)}}{v_{(Q:particle)}} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}$$

$$v_{(P:particle)} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times v_{(Q:Particle)} \qquad (5)$$

So the non-local speed of the particle, $v_{(P:particle)}$, may exceed the standard local value of the speed of light depending on the value of the ratio $g_{tt(Q)} : g_{tt(P)}$

Incidentally the local speed of the particle is always less than the local speed of light, that is

$$v_{(Q:particle)} < c$$

Therefore from relation (5) we have,

$$v_{(P:particle)} < \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times c \tag{6}$$

But the right hand side of relation (6) is the non-local speed of light [see relation (4)]. Therefore

$$v_{(P:particle)} < c_P \, . \tag{7}$$

Thus the non-local speed of the particle is less than the non local speed of light, though the non-local speed of the particle can exceed the local standard speed of light in vacuum depending on the value of the ratio: $g_{tt(Q)} : g_{tt(P)}$.The light ray is *always ahead of the particle* does not matter whether you are concerned with local or non-local observation. *We are not violating relativity in any manner.*
Now the non-local speed of light or some particle is important in deciding the average speed of light coming across a finite interval of space Time of non-local time of travel of a light ray is given by:

$$\begin{aligned} dT &= \frac{dL}{\sqrt{g_{tt}(P)/g_{tt}(A)} \times c} \\ &Time - Taken: \\ T &= \int_B^A \frac{dL}{\sqrt{g_{tt}(P)/g_{tt}(A)} \times c} \end{aligned} \tag{8}$$

The average speed of light for non-local travel across macroscopic distances:

$$\begin{aligned} c_{Average} &= \frac{\int_B^A dL}{\int_B^A \frac{dL}{\sqrt{g_{tt}(P)/g_{tt}(A)} \times c}} \\ &= c \times \frac{\int_B^A dL}{\int_B^A \frac{dL}{\sqrt{g_{tt}(P)/g_{tt}(A)}}} \neq c \end{aligned} \tag{9}$$

So the average speed of light may be different from the local speed “c”(which corresponds to the known value - the speed of light in vacuum).
When a light ray is coming towards an observer across some interval of space he would be more interested in the average speed of light over the interval than the local speed of light local speed of light for various points traversed by the light ray.

SYNCHRONIZATION OF CLOCKS:

For the purpose of synchronization[4] of clocks we take the speed of light constant over large macroscopic distances. It is really justified in view of the fact that the speed of light may change in the non-local sense especially when we are considering sensitive experiments like the OPERA[5] or ICARUS[6]. It would be an interesting reminder for us that the OPERA experiment failed (due to cable fault: loose cable connection) with the condition that the speed of light was taken to be constant with respect to observation stations in disregard of the fact that the light ray traveled over large macroscopic distances in the process of synchronization. The ICARUS experiment succeeded on the basis of the same "aspect" ---the non-local variation of the speed of light was not given a due consideration.

Sample Calculations

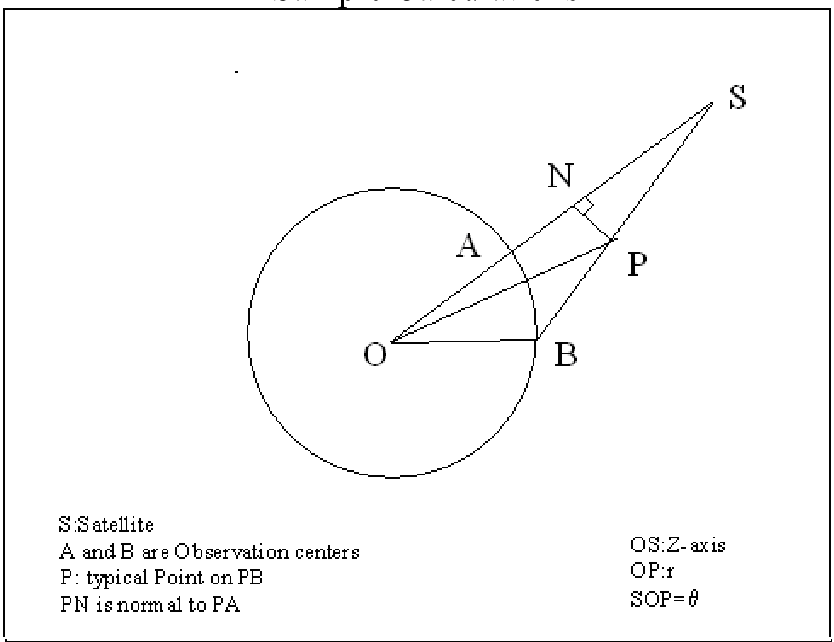

FIGURE 2. Transmission of light ray from a satellite to two earth stations at A and B

The above figure shows a "non-rotating " earth-like planet with observation stations at A and B. S is a satellite from where light signals are being sent. These are being received at the earth stations A and B. O is taken to be the z:axis.OP=r;$\angle$SOP=θ. OS=d, a fixed "coordinate distance".$\angle$OSP=α, a fixed/constant angle. PN is perpendicular to OS.

Now, ON=rCosθ, SN=OS-ON=d-rCosθ,

$$\tan\alpha = \frac{PN}{SN} = \frac{rSin\theta}{d - rCos\theta},$$

$$d\tan\alpha - r\tan\alpha Cos\theta = rSin\theta \tag{10}$$

Taking differentials from (10) we have:

$$dr(Sin\theta + \tan\alpha Cos\theta) = rd\theta(Cos\theta - \tan\alpha Sin\theta) \tag{11}$$

Again from relation (10) we obtain:

$$r = \frac{d\tan\alpha}{Sin\theta + \tan\alpha Cos\theta}\ . \tag{12}$$

Schwarzschild's Metric:

$$ds^2 = (1 - 2GM / c^2 r)dt^2 - (1 - 2GM / c^2 r)^{-1} dr^2 - r^2 (d\theta^2 + Sin^2\theta d\varphi^2)$$

The spatial element on the line SB, say at P, is given by:

$$dL = \sqrt{\left(\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \frac{(Cos\theta - \tan\alpha Sin\theta)^2}{(Sin\theta + \tan\alpha Cos\theta)^2} + 1\right)} \times r d\theta \tag{13}$$

Spatial element on AS is given by:

$$dL = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} dr \tag{14}$$

[Since both α and θ are zero on AS].
Time of travel of light ray from B to S:

$$T = \int_{\theta_S}^{\theta_P} \frac{\sqrt{\left(\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \frac{(Cos\theta - \tan\alpha Sin\theta)^2}{(Sin\theta + \tan\alpha Cos\theta)^2} + 1\right)}}{c \times \sqrt{\frac{\left(1 - \frac{2GM}{c^2 r(P)}\right)}{\left(1 - \frac{2GM}{c^2 r(S)}\right)}}} \times r d\theta \tag{15}$$

"r" may be taken from (12).
Time of light ray from S to A:

$$T = \int_{r_S}^{r_P} \frac{\sqrt{\left(1 - \frac{2GM}{c^2 r}\right)^{-1}}}{c \times \sqrt{\frac{\left(1 - \frac{2GM}{c^2 r(P)}\right)}{\left(1 - \frac{2GM}{c^2 r(S)}\right)}}} \times dr \tag{16}$$

Incidentally, for this path θ and dθ are both zero. So we have considered integration with respect to *dr*.

These calculations take care of the "tick rate" at each point on the path of the light ray while in they GPS they consider the tick rates at the point of transmission and reception only.

NON-LINEARITY OF EINSTEIN'S FIELD EQUATIONS

The fact that Einstein's field equations are non linear is a well known fact in physics. But in the inertial frames of reference the Christoffel symbols[7]evaluate to zero value and the field equations are no more non linear. They become linear. So if you are working in a laboratory you are enjoying the privilege of linearity which is not there outside your laboratory if it(lab) happens to be a local inertial frame. For non local observations the non linearity of the field equations are supposed to play a very big role as in our case of pseudo superluminal motion. One issue becomes important in this respect: to what extent is our lab fixed on the earth's surface is an inertial frame of reference?

Lab Fixed on the Earth's Surface

You are working in your small laboratory room fixed on the earth so that you may call it a local inertial frame[And you are working for a suitably small interval of time]. Now you may think of a freely falling lift in front of you. That lift is a better approximation of a LIF. Your Lab room does not correspond to the "better approximation". The contrast should would become glaringly conspicuous if you imagine the "gravity" to be a million times stronger---that is if you consider your lab room to be in a region of strong spacetime curvature. The freely falling lift is a LIF while your lab room in this example may be termed as a "*Local Non Inertial Frame*". The basic advantage provided by the Local Inertial Frames is the Special Relativity context. The point that naturally arises is that to what extent do we expect deviations from SR in the local non-inertial frame?

The Tangent Plane to the Manifold

Let us consider the tangent plane[8] at the point of contact P with the curved spacetime surface. The tangent surface offers the advantage of the Special Relativity context. *Since time goes on changing in both the tangent plane and the curved surface(though differently)*, our laboratory, its spacetime, location(coordinates) at the most can be at a momentary contact with the point P.Then the space-time point of the laboratory will move along the curved surface unless we make some *technological arrangement* of containing our laboratory on the tangent plane by arranging a freely falling frame.To materialize the local transformation from 4D curved space to Minkowski space we have to arrange a freely falling frame--the falling lift in the simple this case of the earth.

Let the coordinates of the curved 4D surface be $(t,x,y,z)$and the local coordinates on the tangent surface at P: $(\xi_0,\xi_1,\xi_2,\xi_3)$. The first coordinate in parenthesis represents time in each system. If we want to keep the on the tangent surface in order to enjoy the advantage of Special Relativity, the lift should accelerate wrt to the curved surface (generally speaking).

Transformations (Equation Set 17)

$$t=f_1(\xi_0,\xi_1,\xi_2,\xi_3)$$
$$x=f_2(\xi_0,\xi_1,\xi_2,\xi_3)$$

$$y = f_3(\xi_0,\xi_1,\xi_2,\xi_3)$$
$$z = f_4(\xi_0,\xi_1,\xi_2,\xi_3)$$

Our the tangent plane is actually an inertial frame of reference. Consider a world line on it through the point of contact, P a short world line of course . Let us denote the world line by:

$$F(\xi_0,\xi_1,\xi_2,\xi_2) = 0 \tag{18}$$

For the transformed values the quantities , $\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}$ and $\frac{d^2z}{dt^2}$, in general will be non zero.

To understand the situation at the point of contact we consider a simpler analogy. We take the parabola given by: $y = x^2$. Gradient: $\frac{dy}{dx} = 2a$ The tangent to it at the point M(a,b) is given by:

$$\frac{y-b}{x-a} = 2a$$

At the point M(point of contact) the value of $\frac{dy}{dx}$ is identical for both the parabola and the tangent which is a straight line in this case. But what about the second order derivative, $\frac{d^2y}{dx^2}$? For the parabola: $\left[\frac{d^2y}{dx^2}\right]_M = 2$. For the straight line: $\left[\frac{d^2y}{dx^2}\right]_M = 0$. *The second order derivatives differ even at the point of contact.* You may translate this example to higher dimensions.

Points to Observe:

1. The point of contact P on the manifold and the tangent plane are not identical in so far as the second order derivatives are considered. The first order derivatives on the two planes at the point of contact are identical. But they are not identical(in general) at other neighboring points.
2. To stay on the tangent plane[LIF],even at the point of contact ,P, some acceleration is necessary. We need a *freely falling frame* to stay on the said tangent plane.

Speed of Light in Local Inertial Frames

Indeed , we may write the metric:

$$ds^2 = g_{tt}dt^2 - g_{xx}dx^2 \tag{19}$$

$$ds^2 = dT^2 - dL^2 \tag{20}$$

In the above metric, that is in (19), the x-axis has been oriented along the infinitesimal path of a light ray. Now $ds^2 = 0$ for the null geodesic. Therefore from relation (20) we have,

$$Mod\left[\frac{dL}{dT}\right]=1$$

Incidentally c=1 in the natural units and we have the same invariable speed of light in vacuum provided we define physical time interval as: $dT(physical)=\sqrt{g_{tt}}\,dt$

dt is the coordinate time interval.

We are getting the speed of light "c"[standard value] with respect to the tangent plane. Incidentally, equation (20) corresponds to the tangent plane, the LIF.What about equation (19)?It represents curved spacetime. At the point of contact we ,of course, get the same value for the first order derivative for both the surfaces of which the speed of light is an example. But even for short distances this fails—the picture is so tricky even in the contest of the local inertial frames. Pseudo super luminal speed of light in the non-local context is coming into picture! We may try to calculate the acceleration of the light ray at the point of contact of the tangent plane with the manifold(wrt to the manifold).Any deviation from an LIF due to absence of correct amount of acceleration required to stay on the tangent plane will necessitate such an investigation.

ACKNOWLEDGMENTS

I find this an opportunity to express my indebtedness to authors whose works have always inspired me to maintain an active interest in the area in the area of physics. My heartfelt respects go to Dr Timur Kamalov and other persons of the ICTP committee for providing me the opportunity to present the forgoing article at ICTP, 2012, Moscow.

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CRITICAL ANALYSIS OF VELOCITY OF PARTICLE WITH TIME VECTOR POSTULATION

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Abstract. This paper is aimed at critical analysis of velocity measurement of a high velocity particle. In the course of analysis, time dimension is a hypothetic vector dimension and assumed to be perpendicular or independent of rest three space vector dimensions (X,Y,Z) simultaneously, in the form of ct where c is the velocity of any electromagnetic wave or light in vacuum (assumed to be constant in all the reference frames). Analysis shows that when a particle starts moving at a very high velocity with respect to any stationery reference or observer, an error in the measurement is inherently introduced in the measured position and velocity, as velocity of light is finite. Hence, the observed velocity v_0 is correlated with practical velocity v_p. This correlation is used for further interpretation of high velocity relativistic phenomena of particle behavior. Results obtained goes with the particles behavior obtained using special theory of relativity by Prof. Albert Einstein taking all the equations given by him is based on the observed velocity of the particle or reference, not the practical velocity

Keywords: time vector, theory of relativity

INTRODUCTION

Lots of work has been continuously going on the theory of special relativity since last hundred years. Practical verification of the same, in it's different aspects, has established the theory on a mammoth basement. In this paper one basic assumption of special theory of relativity is critically analyzed. When we assume one reference frame is moving at a velocity **v** with respect to other, then space and time dimensions are found to get changed according to theory of special relativity. Now if we critically analyze, it is never possible to determine the position and velocity of a moving object correctly, sensing only the electromagnetic waves coming of it i.e. light. This is due to the fact that light takes some time to reach the observer and when it reaches the observer, particle is not at the same position, as the observer is observing at that instant. Hence there is a difference between the actual velocity or practical velocity of the particle $\mathbf{v_p}$ and the observed velocity $\mathbf{v_0}$ by an observer in the same reference frame. Now if further analysis is done it can be seen that Galelian transformations are valid throughout the range of practical velocity $\mathbf{v_p}$ how much higher it might be. Lorentz's transformation is true with observed velocity $\mathbf{v_0}$**.** The ambiguity or violation of Galelian transformations or general theory of relativity are observed at a very high velocity of particle due to the fact that, observed velocity $\mathbf{v_0}$ is much apart to $\mathbf{v_p}$ at higher velocities. Whereas the above correlations are only true for practical velocities of particles not the observed velocities. At low velocities it can be shown that $\mathbf{v_o} \approx \mathbf{v_p}$. Hence at lower velocities, results go with general theory of relativity but fails at higher velocities.

CONCEPT OF TIME VECTOR

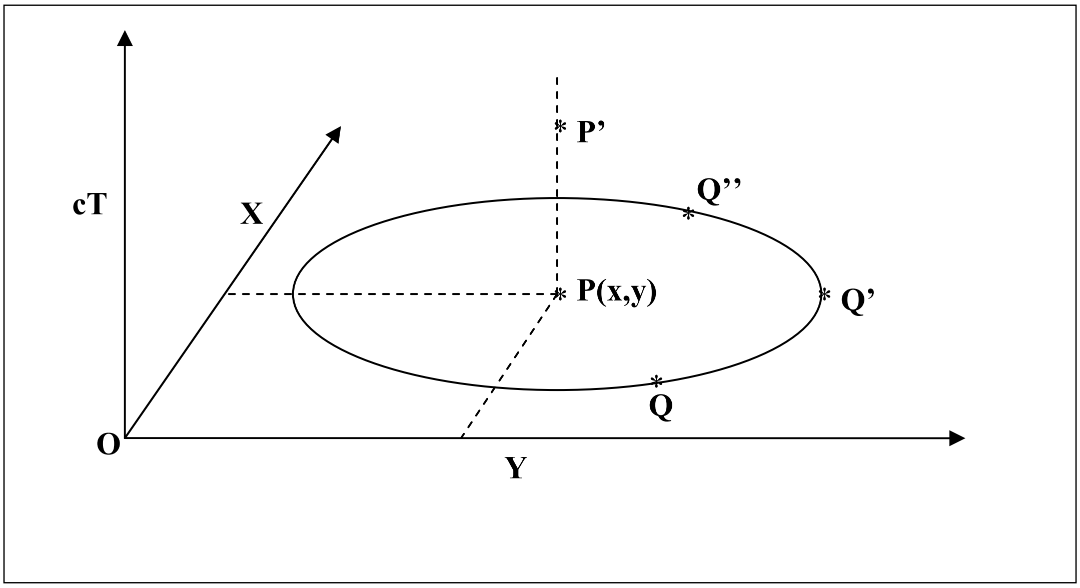

FIGURE 1. Figure showing time vector axis in two dimensional space plane

In determination of the amount of error involved in the measurement of velocities, a new concept of time vector is proposed. Concept of time dimension as a vector is little bit difficult to visualize. In this section this concept is tried to be qualitatively explained.

In space we can define any point by using three independent vectors i.e. X, Y, Z. if we need to define time of any event at that point, time comes as 4^{th} dimension. Now in the concept of time vector, we assume time is another dimensional vector represented as **ct.** The direction of **ct** vector is such that it is independent hence perpendicular to all other three space vectors i.e. X, Y, Z. Physically it is difficult to visualize that four vectors simultaneously perpendicular to each other at a single point. But, this is just to emphasize the physical independence of all the vectors with each other.

For the ease of understanding let us consider a two dimensional plane having two axis X & Y. now if any event takes place at any time t=0 at point P, observers at Q, Q' & Q'' will be getting the information at t = r/c. where r= |PQ|=|PQ'|=|PQ''|. Now if, P point is shifted by an amount of △X or △Y, then all the three points can never see the incident at same time. If the incident takes place at P at $t=t_1$, it physically does imply that all the three points will get the information at some time $t = t_1'$ simultaneously. Now as shifting of the incident is taking place in the time axis, hence it cannot be represented by shifting P point in X or Y direction. Hence we have to shift P point in perpendicular direction to the X-Y plane. This perpendicular direction is defined as T-axis or time axis. And the time at which the event will be observed can be calculated as $t_1'=T. t_1 +R.(r/c)$

Where, T & R represents time vector and radius vector respectively.

In practice the same thing will happen in three dimensional spaces and hence we will get the relation as

$$t' = \vec{t} + (\vec{x}/c) + (\vec{y}/c) + (\vec{z}/c) \quad (1)$$

Where x, y, z are the co-ordinates of point of occurrence, t is the time of occurrence and t' is the time of observation by an observer present at the origin. According to this hypothesis the observation time will not be the algebraic sum of the space and time co-ordinates

$$t' \neq t + |(\vec{x}/c) + (\vec{y}/c) + (\vec{z}/c)| \quad (2)$$

RELATION BETWEEN OBSERVED VELOCITY AND PRACTICAL VELOCITY

Let us now consider that a particle is practically moving at velocity v_p with respect to reference frame. An observer at origin sees the particle to move at a velocity v_0. now if the particle crosses point $P_1(x_1,y_1,z_1)$ at time t_1 and reaches $P_2(x_2,y_2,z_2)$ at time t_2 then observer at origin will see the particle to cross P_1 at t_1' and P_2 at t_2'.

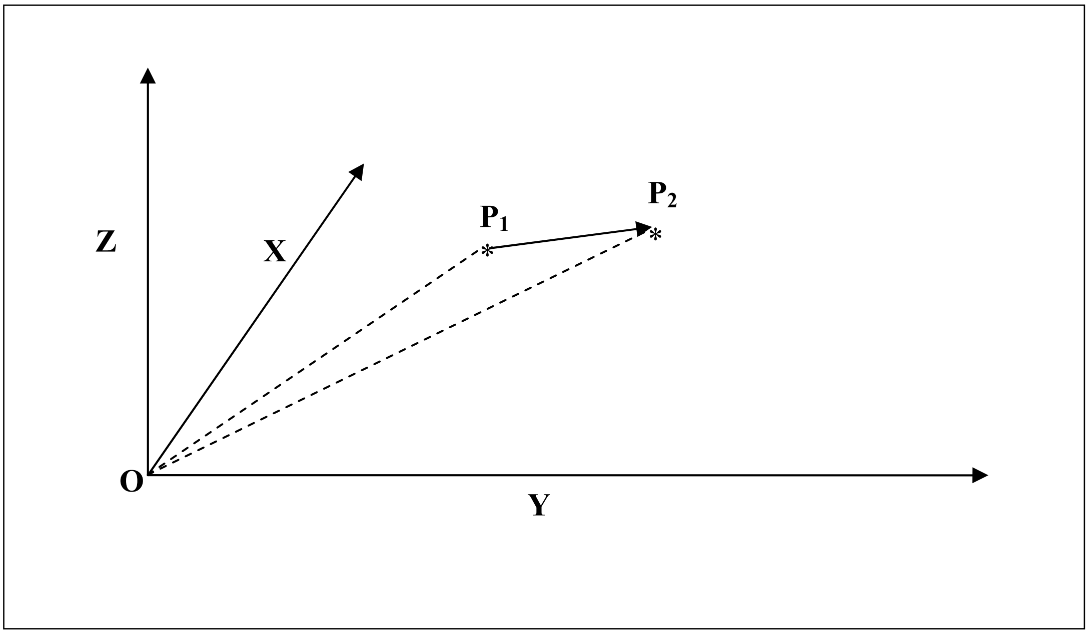

FIGURE 2. Figure showing particle moving from P_1 to P_2 and being observed from origin

Now, according to equation (1)

$$t_1' - t_2' = \overrightarrow{t_1 - t_2} + \overrightarrow{(x_1 - x_2)/c} + \overrightarrow{(y_1 - y_2)/c} + \overrightarrow{(z_1 - z_2)/c} \tag{3}$$

Therefore,

$$(t_1' - t_2')^2 = (t_1 - t_2)^2 + (x_1 - x_2)^2/c^2 + (y_1 - y_2)^2/c^2 + (z_1 - z_2)^2/c^2$$

Or, $(t_1' - t_2')^2 = (t_1 - t_2)^2 + l^2/c^2$

Where, l= P_1P_2

As practical velocity $v_p = l/(t_1-t_2)$ and observed velocity $v_0 = l/(t_1'-t_2')$

We can rewrite the equation as

$$l^2/v_0^2 = l^2/v_p^2 + l^2/c^2$$

Or,

$$1/v_0^2 = 1/v_p^2 + 1/c^2 \tag{4}$$

Now, the equation can be re-written as

$$v_0 = v_p c / \sqrt{(v_p^2 + c^2)} \tag{5}$$

Hence if $v_p \rightarrow \alpha$, $v_0 \rightarrow c$. therefore if practical velocity of particle tends to infinity, observed velocity v_0 will tend to c. Therefore, it is never possible to see any particle to move more than the velocity of light. But if $v_0 > c/\sqrt{2}$, then definitely the particle is moving beyond velocity of light i.e. $v_p > c$, but we cannot experience, feel or sense that as we try to see it using electromagnetic waves only.

CORRELATION WITH SPECIAL THEORY OF RELATIVITY

Let us now consider how dimensional change of any system can be correlated with the conclusions of the previous section. In the previous section we have seen that there is a particular relation exists between the practical velocity and observed velocity of any system. Now if any reference system is found to move with a relative velocity of v_0 with a stationery reference frame, observed by an observer present at the origin of the stationery reference frame. Then the system is practically moving at a velocity of v_p with respect to the stationery reference frame. Now, if we integrate both sides of equation (5) then for time independent velocity we will get

$$x_0 = x_p c / \sqrt{(v_p^2 + c^2)} \tag{6}$$

Where, $x_0 = v_0.t$ and $x_p = v_p.t$

Therefore now it is seen that any point on the stationery reference frame if observed by the moving reference frame to be at a distance of x_0, then practically it is at a distance of x_p. But, as there is no movement of the observing point with respect to stationery reference frame hence distance observed by the stationery reference frame will be x_p only. Hence we can conclude that any distance x_p in stationery reference frame while observed from a moving reference frame it will be observed as x_0. Therefore, if any point is at x at t=0 from stationery reference frame, and assuming both the stationery and moving reference frames coinciding at there origin at t= 0, then at any time t the observed distance between moving reference frame and the point will be ($x-v_0.t$) by the moving reference and the distance between them observed by the stationery reference frame will be

($x-v_0.t$).$c\sqrt{(v_p^2 + c^2)}$.

Therefore, we can say that

$$x' = \frac{(x-v_0t)c}{\sqrt{(v_p^2+c^2)}} = \frac{(x-v_0t)c}{\sqrt{(c^2-v_0^2)}} \tag{7}$$

Similarly, if we do the analysis of path traveled by light at t time we can show that

$$ct' = \frac{\left(ct - v_0 \frac{x}{c}\right)c}{\sqrt{(v_p^2+c^2)}} = \frac{\left(ct - v_0 \frac{x}{c}\right)c}{\sqrt{(c^2-v_0^2)}}$$

Or,

$$t' = \frac{\left(t - v_0 \frac{x}{c^2}\right)c}{\sqrt{(v_p^2+c^2)}} = \frac{\left(t - v_0 \frac{x}{c^2}\right)c}{\sqrt{(c^2-v_0^2)}} \tag{8}$$

where, x' and t' denotes distance and time, of any point incident (not moving in the stationary reference frame) and a moving reference frame of observed velocity v_0 with respect to stationery reference frame in x direction, as seen by an observer at the stationery reference frame origin.

The above equations are known previously as Lorentz's transformation. Let us now represent these equations in different form and analyze their physical implications. Hence, equation (7) and (8) are rewritten as

$$x' = (x - v_0t)\frac{v_p}{v_0} \tag{9}$$

$$t' == \left(t - v_0 \frac{x}{c^2}\right)\frac{v_p}{v_0} \tag{10}$$

Now, observing equation (9) and (10) we can say that change in the observed dimensions due to special relativity is a scale change by a factor of v_p/v_0 . The factor is coming into the picture due to observation error involved in the determination of position, velocity and instant of position of any particle due to its high velocity with respect to the reference frame. If we could have observed practical velocity of any high velocity particle more accurately, the factor would have been reduced to 1 and there would not have been any special relativistic effect.

CALCULATION OF KINETIC ENERGY OF HIGH VELOCITY PARTICLES

If we conclude from the previous section, that all the dimensional changes are observation errors, hence mass and energy of the particle should not also change relativistically. But in practice they do follow the laws of special theory of relativity. Now in this section this paradox is tried to be explained.

According to special theory of relativity, velocity change of particle incorporates with change in mass. If any particle is tried to be accelerated continuously, its energy increment not only takes place in the form of velocity increment only but mass increment also.

Let us now consider that no mass change takes place with change in velocity of the particle. And thereafter, let us calculate effect of constant force and gained kinetic energy of an accelerating particle.

If we assume a continuous accelerating force F is applied to a particle to accelerate it, then the amount of energy stored in the particle after traveling an observed distance of x_0 is given by the following equation. Here it can be proved that, if we consider particles observed velocity $\mathbf{v_0}$ then consideration of change of mass with respect to velocity is to be considered. But, if we consider practical velocity of particle i.e. $\mathbf{v_p}$, then no change in mass is required to be considered. This is because of the fact that observed velocity is an erroneous observation and does not go with Newton's laws of motion. To compensate for the error we consider mass of the particle to get change. But if we directly consider practical velocity of the particle no such mass change compensation is required.

$$F = d(m_0.v_P) = m_0 dv_P = m_0.d\left(\frac{v_0}{\sqrt{\left(1-{v_0^2}\big/{c^2}\right)}}\right) = \frac{m_0}{\sqrt{\left(1-{v_0^2}\big/{c^2}\right)}}.dv_0 + v_0 d\left(\frac{m_0}{\sqrt{\left(1-{v_0^2}\big/{c^2}\right)}}\right) \tag{11}$$

$$F = m.dv_0 + v_0.dm.$$

$$E = \int_0^{x_0} F.dx_0 = \int_0^{x_0} d\left(m_0 v_p\right)dx_0 = \int_0^{v_P} m_0 v_0 dv_p \tag{12}$$

While observed at any position, the observed distance and velocity of the particle has to be considered. Whereas change in velocity due to application of force, will take place in consideration with practicals' velocity of the particle. The above equation (12) is formed keeping this physical concept in mind.

Now, if the integration is done using correlation between v_0 and v_p then we will get energy expression as following.

$$E = \int_0^{v_P} m_0 v_0 dv_p = \int_0^{v_0} m_0 v_0 \left(\frac{v_p}{v_0}\right)^3 dv_0 = \int_0^{v_0} \frac{m_0 v_0 dv_0}{\left(1-\frac{v_0^2}{c^2}\right)^{3/2}} \tag{13}$$

$$E = m_0 c^2\left[\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} - 1\right] = m_0 c^2\left[\sqrt{1+\frac{v_p^2}{c^2}} - 1\right] \tag{14}$$

Hence, the amount of kinetic energy gained by any moving particle at any observed distance x_0 is given by equation (14). The calculation shows that it has nothing to do with change in mass. And hereby it is concluded that mass never changes with velocity of any particle. Momentum balance of any inertial frame can also be verified, in the similar fashion, considering the particle's practical velocity and observed velocity properly as required. And then the postulation of change in mass of the particle will not be required to consider.

CONCLUSION

In the above sections different aspects of critical analysis of high velocity particles are presented. Firstly, vector property of time is postulated and physical phenomena or practical implications of observed dimensional changes due to high velocity of reference frame are critically analyzed. Again it has been shown that change in dimensions as observed from high velocity reference frame is an observation error only. The calculation of moving particle's kinetic energy is done using the postulated concept and no change in mass of the particle due to high velocity is proved. Whatever be our reference frame, we always see erroneous time and space co-ordinates of any moving particle. Special theory of relativity tells about any system as sensed or observed from any reference. But, specifying practical position and velocity of any particle is out of the scope of that theory. This paper enlightens quantitatively on erroneous observations and consequences. Further analysis and experiments can be proposed to prove this special vector property of time. But, derivations and explanations of well established previous theories on the basis of the given hypothesis, that time is a vector, establishes the basic postulation to a good extent.

Quantum Space and One Method of Deformation Quantization

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Abstract. Axioms of quantum field theory [1,8] show that the global system of elementary particles can be represented as co- pre- sheaf of C^*- algebras on the physical space. From the properties of C^*-algebra [5,12] it is obvious that this co-pre- sheaf is a pre-sheaf as well [9,10]. If you see those characteristics of the global system of elementary particles that determine the location of individual particles in physical space, then the space of pure states PU of the C^*- algebra of these observables will be the physical space. Each elementary particle (elementary field) in this space is a pair: $(O,\Gamma(O))$, where $O \in PU$ is an open set in the space of pure states PU of the C^*- algebra U of observables of the global system of elementary particles, $\Gamma(O)$ is C^*- sub algebra of C^*- algebra U corresponding to the O . We can assume that the system of elementary particles $(O_1,\Gamma(O_1))$, $(O_2,\Gamma(O_2)),\ldots, (O_n,\Gamma(O_n))$, is in the n dimensional interaction, if $\bigcap_{i=1}^{n} O_i \neq \Theta$. Naturally, the system of elementary particles being in such interaction, consider a single particle. Let $\{(O_\alpha,\Gamma(O_\alpha))\}$ be such a system of particles, which $\{O_\alpha\}$ is an open covering of physical space. In this case, the graded C^*- Algebras: chain complex with coefficients in the co- pre- sheaf $\Im_U = \{\Gamma_C(O)_{O\subset PU}\}$ [9,10,13] and the co chain complex with coefficients in the pre- sheaf $\Im_U = \{\Gamma(O)_{O\in PU}\}$ [9,1013] represents the interactions of elementary particles in the system $\{(O_\alpha,\Gamma(O_\alpha))\}$. We can also assume that the boundary and the co-boundary operators of these graded C^*- algebras are the annihilation and creation operators, respectively. When U is a commutative C^* - algebra, the corresponding physical space is a classic, but when the U is a non-commutative C^*- algebra, corresponding physical space is a quantum [4]. In applying the theory of universal representations of C* - algebras constructed a continuous deformation of non-commutative C*-algebra to a commutative C* - algebra, which is called "a deformation quantization"[4]. Classical and quantum physical spaces, resulting by such construction of deformation quantization are homotopy equivalent.

Keywords: C^*- algebra, co-pre- sheaf , pre- sheaf , chain complex, co- chain complex, annihilation operator, creation operator, n - dimensional interaction, deformation quantization, classical spaces, quantum spaces, gravitational collapse, black hole .

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1. STRUCTURAL PRE-SHEAF OF C^*-ALGEBRA

Let U be a C^*-algebra, FU be s set of states on this algebra [5,12]. Consider the universal representation $\pi_U : U \to \Im(H^U)$ of the algebra U [5,12], where $H^U = \bigoplus_{f \in FU} H^f$ and H^f is the Hilbert space associated with a state f. As we know, a universal representation is an exact one, it is an isometric embedding.

Let $U' \subset U$ be a sub algebra of the C^*-algebra U. According to the Khan-Banach theorem, the mapping $Q : FU \to FU'$ defined by the formula $Q(f) = f|_{U'}$ is surjective.

Consider the Hilbert space $H^U = \bigoplus_{f|_{U'} \in FU'} H^{f|_{U'}}$. Clearly, there exists an isometric embedding $I_{U'} : H^{U'} \to H^U$ and therefore it can be assumed that $H^{U'} \subset H^U$ Consider the projector $P_{U'} : H^U \to H^{U'}$. Let $A \in \Im(H^U)$, then $P_{U'} \circ A \circ I_{U'} \in \Im(H^{U'})$ and we have the *-morphism $T_U : \Im(H^U) \to \Im(H^{U'})$ defined by the formula $T_U(A) = P_{U'} \circ A \circ I_{U'}$.

If $\pi_{U'} : U' \to \Im(H^{U'})$ is a universal representation, then we assume that $\tau_{U'} : U \to U'$ is the mapping defined by the formula $\tau_{U'} = \pi_{U'}^{-1} \circ T_U \circ \pi_U : U \to U'$. It is well-defined because if $A \in \pi_U(U)$, then from the construction of the universal representation it follows that $P_{U'} \circ A \circ I_{U'} \in \pi_{U'}(U')$, which means that $\tau_{U'} : U \to U'$ is well-defined.

Let $\Pr imU$ be the space of primitive ideals of the C^* -algebra U. with Jacobson topology [5,12]. The elements of this space are the kernels of irreducible representations of the C^*-algebra U. It is obvious that these kernels are bilateral ideals. Let us assume that $U' \subset U'$. Consider those irreducible representation π^U which have the form $\pi^U = \pi^{U'} \circ \tau_{U'}$, where $\pi^{U'} : U' \to \Im(H')$. The set of kernels of such representations is a subset of the space $\Pr imU$.

Let us now consider the space $\Pr imU'$ and the mapping $S : \Pr imU' \to \Pr imU$ defined by the formula $S(\beta) = \tau_{U'}^{-1}(\beta); \beta \in \Pr im, \beta = \ker \pi^U$. This mapping is an injection since if

$$\beta_1, \beta_2 \in \Pr imU'; \beta_1 \neq \beta_2, a \in \beta_1, a \notin \beta_2,\ \tau_{U'}^{-1}(a) \in \tau_{U'}^{-1}(\beta_1) = \tau_{U'}^{-1}(\beta_2),$$

then $a \in \beta_2$, which is impossible. Thus our assumption is not true, $\beta_1 = \beta_2$ and we can write $\Pr imU' \subset \Pr imU$.

Let us assume that $O \in \Pr imU$ is an open set. It is a complement of the set F of primitive ideals which contain some fixed subset $M \subset U$. Thus $O = \Pr imU \setminus F$ consists of primitive ideals U of the algebra which do not contain the subset $M \subset U$. The closure $\overline{O}$ is the set of primitive ideals which contain some subset

$M' \subset U; M' \neq M$ of the algebra U . The set $P \cap \overline{O}$ consists of primitive ideals which contain both the set M and the set M'The set $\overline{O} \mid (F \cap \overline{O}) = O$ consists of primitive ideals which contain the set M' and do not contain the set M .

Let $\alpha \in O$, then $\alpha = \ker \pi^U$, where $\pi^U : U \to \Im(H)$ is some irreducible representation. Consider the factor-algebra U/U', where $U' = \bigcap_{\alpha \in O} \alpha$. If $p : U \to U/U'$ is a canonical projection and $\pi^{U/U'} : U/U' \to \Im(H)$ is some irreducible representation, then the surjectivity of p implies that the representation $\pi^U_{U'} = \pi^{U/U'} \circ p$ is irreducible. It is obvious that $M' \subset \ker \pi^U_{U'}$. Hence it follows that $\ker \pi^U_{U'} \in \overline{O}$.

Consider $\alpha \in O$ and the representation $\pi_\alpha \circ \overline{p} \circ p : U \to \Im(H^{U/\alpha})$, where $\overline{p} : U/U' \to U/\alpha$ is defined by the formula $\overline{p}([a]) = [[a]]$, the class $[[a]]$ is obtained by the equivalence relation $[a] \approx [b]$ if $a - b \in \alpha$, $\pi_\alpha : U/\alpha \to \Im(H^{U/\alpha})$ is the irreducible injective representation of the primitive algebra U/α .

Since $\overline{p} \circ p : U \to U/\alpha$ is a canonical projection, we have $\ker(\pi_\alpha \circ \overline{p} \circ p) = \alpha$. Therefore if $\pi^{U/U'} = \pi_\alpha \overline{p}$, then $\ker \pi^U_{U'} = \alpha$ and the set of primitive ideals of the algebra U/U' coincides with the set O.

Let $\pi^{U/U'} : U/U' \to \Im(H)$ be some irreducible representation. We consider the representat $\pi^U_{U'} = \pi^{U/U'} \circ p$ and the kernel of this representation $\ker \pi^U_{U'} = p^{-1}(\ker \pi^{U/U'})$.

The factor-algebra $U/p^{-1}(\ker \pi^{U/U'})$ is primitive. The C^*-morphism $\pi_{p^{-1}(\ker \pi^{U/U'})} : U/p^{-1}(\ker \pi^{U/U'}) \to \Im(H)$ defined by the formula $\pi_{p^{-1}(\ker \pi^{U/U'})}([[a]]) = \pi^{U/U'}([a])$ is an irreducible injective representation. Thus $\pi^U_{U'} = \pi_{p^{-1}(\ker \pi^{U/U'})} \circ \overline{p} \circ p$ and hence it follows that each representation $\pi^U_{U'} = \pi^{U/U'} \circ p$ has the form $\pi^{U'}_{U'} = \pi_\alpha \circ \overline{p} \circ p$ and its kernel is equal to $\alpha \in O$.

From the above reasoning we have $O = \Pr im U/U'$, where $U' = \bigcap_{\alpha \in O} \alpha$.

Since U is a C^*-algebra, for each bilateral ideal $I \subset U$ the algebra U/I is a primitive module [5,12]. Hence it follows that there exists a continuous C^*-monomorphism $\varphi : U/I \to U$, for which the equality $p \circ \varphi = id_{U/I}$ is fulfilled. Therefore it can be assumed that $U/U' \subset U$.

As we see, to each set $O \subset \Pr imU$ there corresponds a C^*-algebra $U_O \cong U/U'; U' = \bigcap_{\alpha \in O} \alpha$ such that $O = \Pr imU_O$. If $O_1 \subset O_2 \subset \Pr imU$, then $U_1' \subset U_2'$ and there exists a C^*-morphism of the algebra $U_{O_2} \cong U/U_2'; U_2' = \bigcap_{\alpha \in O_2} \alpha$ into the algebra $U_{O_1} \cong U/U_1'; U_1' = \bigcap_{\alpha \in O_1} \alpha$. We denote this C^*-morphism as follows: $\rho_{O_1}^{O_2} : U_{O_2} \to U_{O_1}$. Clearly, if $O_1 \subset O_2 \subset O_3 \subset \Pr imU$, then $\rho_{O_1}^{O_2} \circ \rho_{O_2}^{O_3} = \rho_{O_1}^{O_3}$ and $\rho_O^O = Id_{U_O}$.

If, additionally, to the empty set $\Theta \subset \Pr imU$ we put into correspondence a trivial C^*-algebra, then on the space $\Pr imU$ we will define the structural pre-sheaf of C^*-algebras [9,10]. We denote it by $\Im_U$.

The pre-sheaf $\Im_U$ has the following property: for any open subsets $O' \subset O \subset \Pr imU$ we have $U_{O'} \subset U_O$. This means that $\Im_U$ is a co-pre-sheaf as well [9, 10]. We denote it by $\Im_U^C$.

Let $SpecU$ be the spectrum of a C^*-algebra U. There exists a surjection $\phi : SpecU \to \Pr imU$ [6]. It defines a topology on the set $SpecU$. This is the weakest topology among such topologies on $SpecU$, for which ϕ is continuous.

Let now PU be the set of all pure states on U [1]. There exists a surjection $\varphi : PU \to SpecU$ [5] which also defines a topology on the set PU. This is the weakest topology among such topologies on PU, for which φ is continuous when on $SpecU$ we have the above-mentioned topology.

Topological structures on the spaces PU, $SpecU$ and $\Pr imU$ are not always good (in the sense of separability), but we think that the construction of such structures makes sense all the same .

The mappings $\phi, \varphi, h = \varphi \circ \phi$ are open and therefore the structures of a pre-sheaf and a co-pre-sheaf can be extended from the space $\Pr imU$ to PU and $SpecU$.

2. A ONE METHOD OF DEFORMATION QUANTIZATION

Let us consider some option of deformation quantization. Let U be any C^*-algebra, while M^U C^*-algebra of the corresponding operators at the universal representation of Algebra U in relevant Hilbert space H^U. Let M be the C^*-algebra of all operators of Hilbert space H^U. Consider the C^*-Algebra $M_{[[t]]}$ of convergent series $A_0 + A_1 A_t + A_2 A_t^2 + A_3 A_t^3 + \dots$ with coefficients in M^U where operator $A_t \in M^U$ defined as: $A_t(v) = tv,\ t \in C$. Let us denote sub C^*-algebra of series which zero degree members are element of M^U thus: $\overline{M_{[[t]]}^U}$. For each fixed variable $t = t_0$ from $[0, h) \subset R$ and operators $A_0 + A_1 A_{t_0} + A_2 A_{t_0}^2 + A_3 A_{t_0}^3 + \dots$ formed C^*-algebra. Denote such C^*-algebra of operators us follows: M_{t_0}. It is clear, that $M_t \to M_0 = M^U \approx U$

when $t \to 0$.So we build a continuous process, which C^*-algebra U_t moves to C^*-Algebra U. Such continuous process is called deformation. If the C^*-algebra U is commutative , then process of such deformation process in Mathematical physics is called “deformation quantization” [4].

Theorem 1. $M^U \cong M_t \cong \overline{M}_{[[t]]}$, where the symbol $\cong$ denotes the homotopy equivalence.

Proof: It is clear , $M_t \subset \overline{M}_{[[t]]}$ for all $t \in [0,h)$ and has place a C^*- epymorphizm: $\varphi : \overline{M}_{[[t]]} \to M^U, \varphi(A_0 + A_1 A_t + A_2 A_t^2 + ...) = A_0$, where $A_0 \in M^U$. It is also clear that: $\varphi^{-1}(A_0) = \varphi^{-1}(A_0') = Ker\varphi \ = \{A_1 A_t + A_2 A_t^2 + A_3 A_t^3 + ...\}$ for all $A_0, A_0' \in M^U$ couples of elements.

There exist section: $s_\varphi : M^U = \overline{M}_{[[t]]} / Ker\varphi \to \overline{M}_{[[t]]}$ which meets the condition $\varphi \circ s_\varphi = id_{M^U}$. Since $\varphi^{-1}(A_0) = \varphi^{-1}(A_0') = Ker\varphi \ = \{A_1 t + A_2 A_t^2 + A_3 A_t^3 + ...\}$ is C^*-algebra, it is convex and therefore contractible . This means, that $s_\varphi \circ \varphi \cong id_{\overline{M}_{[[t]]}}$ and algebras M^U and $\overline{M}_{[[t]]}$ are homotopy equivalent.

Let $t = t_0$, then $M_{t_0} = \overline{M}_{[[t]]} / Ker\gamma \to \overline{M}_{[[t]]}$ where $\gamma : \overline{M}_{[[t]]} \to M^U$ C^*-morphism, which is defined by the formula: $\gamma(A_0 + A_1 A_t + A_2 A_t^2 + ...) = A_0 + A_1 A_{t_0} + A_2 A_{t_0}^2 + ...$, $Ker\gamma = \{A_0 + A_1 A_t + A_2 A_t^2 + ... \mid A_0 + A_1 A_{t_0} + A_2 A_{t_0}^2 + ... = 0\}$.

It is clear that $\gamma^{-1}(A_0 + A_1 A_{t_0} + A_2 A_{t_0}^0 + ...) = \{A_0 + A_1 A_t + A_2 A_t^2 + ...\}_{t \in [0,h)}$, is convex, therefore contractible. So C^* - algebra M_{t_0} homotopy equivalent to C^* -algebra $\overline{M}_{[[t]]}$. Finally, we have that for any $t = t_0 \in [0,h)$ numbers C^* - algebra M^U homotopy equivalent to C^* - algebra M_{t_0}. So, finally we have: $M^U \cong M_{t_0} \cong \overline{M}_{[[t]]}$ q.e.d.

From this theorem it follows that: $\Pr im\overline{M}_{[[t]]} \cong \Pr imM^U$.

Let $\alpha \subset M^U$ be a primitive ideal, which corresponds to irreducible representation π_{M^U} of the M^U algebra in Hilbert space. Let us associate to this primitive ideal α primitive ideal $\beta \subset \overline{M}_{[[t]]}$ the corresponding irreducible representation $\pi_U \circ \varphi$ of the algebra $\overline{M}_{[[t]]}$. So we build a continuous map $\varphi^* : \Pr imM^U \to \Pr im\overline{M}_{[[t]]}$ for Jacobson topological structures. Here, let $\beta \subset \overline{M}_{[[t]]}$ be any primitive ideal which corresponds to irreducible presentation $\pi_{\overline{M}_{[[t]]}}$ in Hilbert space. Let us associate to this primitive ideal the primitive ideal $\alpha' \subset M^U$ defined by irreducible representation $\pi_{\overline{M}_{[[t]]}} \circ s_\varphi$. Thus, we have a continuous map:

$s_{\varphi}^{*}:\Pr im\overline{M}_{[[t]]}\to \Pr imM^{U}$. It's easy to see, that $\varphi^{*}\circ s_{\varphi}^{*}\cong id_{\Pr im\overline{M}_{[[t]]}}$ and $s_{\varphi}^{*}\circ\varphi^{*}\cong id_{\overline{MC}_{[[t]]}}$. Therefore, $\Pr im\overline{M}_{[[t]]}\cong \Pr imM^{U}$.

3. CLASICAL AND QUANTUM PHYSICAL SPACE AND ELEMENTARY PARTICLE SYSTEMS

According to axioms of Algebraic quantum field theory [1,4,8] and properties of C^{*}- Algebra the global system of Elementary Particles (elementary field) may represent as $\Im_{U}^{C}=\{\Gamma_{C}(O)_{O\in PU}\}$ co-pre- shaves and $\Im_{U}=\{\Gamma(O)_{O\in PU}\}$ pre-shaves of C^{*}- algebras defined by some C^{*} - algebras on the space of pure states PU. As we mentioned above, $\Gamma_{C}(O)=\Gamma(O)$, the space PU, in this case, represents the physical space: the classical one, if C^{*}-algebra U commutative, and the quantum one if C^{*}- algebra U non commutative. Each elementary particle (elementary fields) in this space is a pair: $(O,\Gamma(O))$, where $O\subset PU$ open sub set. We can conclude, that elements of elementary particles system: $(O_1,\Gamma(O_1))$, $(O_2,\Gamma(O_2))$,..., $(O_n,\Gamma(O_n))$ are present in the n- dimensional interaction, if $\bigcap_{i=1}^{n}O_i\neq\Theta$.

Naturally, such a system of elementary particles may be considered as a single particle. Suppose $\{(O_\alpha,\Gamma(O_\alpha))\}$ is a particles system, such that $\Omega=\{O_\alpha\}$ represents the open cover of the physical space PU. We can conclude that the chain complex with coefficients in $\Im_{U}^{C}=\{\Gamma_{C}(O)_{O\in PU}\}$ co-pre-shaves and co-chain complex with coefficients in $\Im_{U}=\{\Gamma(O)_{O\in PU}\}$ pre-shaves :

$$C_k(\Omega,\Im_U^C)=\prod_{j_0 j_1\cdots j_k}\Gamma_C(O_{j_0}\cap O_{j_1}\cap\ldots\cap O_{j_k});\partial_k: C_k(\Omega,\Im_U^C)\to C_{k-1}(\Omega,\Im_U^C);$$

$$(\partial_k f)_{j_0 j_1\cdots j_{k-1}}=\sum_{\omega}\sum_{s=0}^{k}(-1)^s r^{O_{j_0}\cap O_{j_1}\cap\ldots\cap O_{j_{k-1}}\cdot}_{O_{j_o}\cap O_{j_1}\cap\ldots\cap O_{j_{s-1}}\cap O_\theta\cap\ldots\cap O_{j_{k-1}}}(f_{j_0 j_1\cdots j_{s-1}\ \omega\ j_s\cdots j_{k-1}}),$$

$$C^k(\Omega,\Im_U)=\prod_{j_0 j_1\cdots j_k}\Gamma(O_{j_0}\cap O_{j_1}\cap\ldots\cap O_{j_k});\delta^k: C^k(\Omega,\Im_U)\to C^{k+1}(\Omega,\Im_U);$$

$$(\delta^k f)_{j_0 j_1\cdots j_{k+1}}=\sum_{s=0}^{k+1}(-1)^s\rho^{O_{j_0}\cap O_{j_1}\cap\ldots\cap O_{j_{s-1}}\cap\hat{O}_{j_s}\cap O_{j_{s+1}}\cap\ldots\cap O_{j_{k+1}}}_{O_{j_0}\cap O_{j_1}\cap\ldots\cap O_{j_{k+1}}}(f_{j_0 j_1\cdots j_{s-1}\hat{j}_s j_{s+1}\cdots j_{k+1}}).$$

$C_k(\Omega,\Im_U^C)=0,\ C^k(\Omega,\Im_U)=0$, if $k<0$ particle interaction in the system $\{(O_\alpha,\Gamma(O_\alpha))\}$ described.

Moreover, we can conclude that the boundary and co boundary operators of this graded module represent, well known in quantum field theory, annihilation and creation operators,respectively. Co-chain complexes: $C^k(\Omega,\Im_U=(\Gamma(O)_{O\in PU}),\delta^k)$ ar egraded C^{*}-algebras, indeed, define a multiplication operation of the module $\sum_{k\in Z}C^k(\Omega,\Im_U)$. Let $\{f_{i_0 i_1\ldots t_l}\}\in C^l(\Omega,\Im_U),\ \{f_{i_0 i_1\ldots i_j}\}\in C^j(\Omega,\Im_U),$ They can be multiplied:

$$\{f_{i_0^1 i_1^1\ldots t_l^1}\}\cdot\{f_{i_0^2 i_1^2\ldots i_j^2}\}=\{\rho^{O_{i_0^1}\cap O_{i_1^1}\cap\ldots\cap O_{i_l^1}\cap O_{i_0^2}\cap O_{i_1^2}\cap\ldots\cap O_{i_j^2}}_{O_{i_0^1}\cap O_{i_1^1}\cap\ldots\cap O_{i_l^1}}(f_{i_0^1 i_1^1\ldots t_l^1})\cdot$$

$$\cdot \rho_{O_{i_0^2} \cap O_{i_1^2} \cap \ldots \cap O_{i_l^2}}^{O_{i_0^1} \cap O_{i_1^1} \cap \ldots \cap O_{i_l^1} \cap O_{i_0^2} \cap O_{i_1^2} \cap \ldots \cap O_{i_j^2}} (f_{i_0^2 i_1^2 \ldots i_j^2})\} \in C^{l+j}(\Omega, \Im_U)$$
.

It is easy to show that co-boundary operator of our complex correspond to this multiply:

$$\delta^k(a \cdot b) = \delta^l a \cdot b + a \cdot \delta^j b$$
.

Therefore, for such operation $\sum_{k \in Z} C^k(\Omega, \Im_U)$ is graded C^*-algebra. This means that the

$$C^k(\Omega, \Im_U = (\Gamma(O)_{O \in PU}), \delta^k)$$

is graded C^* Algebra.

Back to the covering $\Omega = \{O_\alpha\}$, if we consider this family as pre-basic, in the set PU we obtained topological structure with pre-basic $\Sigma = \{\cup_\beta \Omega_\beta\}$, where $\{\Omega_\beta\}$ is family of open coverings inscribed in covers Ω. Denote the resulting space as PU_Ω. Evidently,

$$PU_\Omega = P \sum_{k \in Z} C^k(\Omega, \Im_U)$$
.

Let $\Xi = \{\Omega\}$ is a complete family of open coverings of space PU. Knowing [13] how homomorphisms ${}^k\theta_\Omega^{\Omega'} : C^k(\Omega, \Im_U) \to C^k(\Omega', \Im_U)$ form a direct spectrum of modules it is clear that $\theta_\Omega^{\Omega'}(\{(f)_{i_0 i_1 \ldots i_k}\}) \cdot \theta_\Omega^{\Omega'}(\{(g)_{j_0 j_1 \ldots j_p}\}) = \theta_\Omega^{\Omega'}(\{f_{i_0 i_1 \ldots i_k}\} \cdot \{g_{j_0 j_1 \ldots j_p}\})$, where $\theta_\Omega^{\Omega'} = \{\sum_k {}^k\theta_\Omega^{\Omega'}\}_{k \in Z}$. So that $(\sum_{k \in Z} C^k(\Omega, \Im_U),\ \theta_\Omega^{\Omega'},\ \{\Omega\})$ is a direct spectrum of C^* algebras. The limit of the direct spectrum of C^*- algebras is C^*-algebra[11]. This direct spectrum corresponds to the direct line spectrum of topological spaces $(PU_\Omega,\ \iota_\Omega^{\Omega'},\ \{\Omega\})$, where $\iota_\Omega^{\Omega'} : PU_\Omega \to PU_{\Omega'}$ are identical maps so, it is clear that it is continuous. It is also obvious that the limit of this spectrum is PU.

The topological structure in space is such that space PU is homotopy equivalent to $\Pr imU$ space. Because $\Im_U^C = \{\Gamma_C(O)_{O \in PU}\}$ co-preshaves and $\Im_U = \{\Gamma(O)_{O \in PU}\}$ pre- shaves of C^*-algebras are induced from $\Im_U^C = \{\Gamma_C(O)_{O \in \Pr imU}\}$ co-pre shaves and $\Im_U = \{\Gamma(O)_{O \in \Pr imU}\}$ preshaves, respectively. On this account, chain and co-chain complexes:

$$C_k(\Omega, \Im_U^C = (\Gamma(O)_{O \in PU}), \partial_k),\ C^k(\Omega, \Im_U = (\Gamma(O)_{O \in PU}), \delta^k)$$

are chain equivalent to chain and co-chain complexe $C_k(\Omega, \Im_U^C = (\Gamma(O)_{O \in \Pr imU}), \partial_k)$, $C^k(\Omega, \Im_U = (\Gamma(O)_{O \in \Pr imU}), \delta^k)$ respectively.

For our specific deformation quantization quantum and classical physical spaces are equivalent up to homotopy equivalence. Therefore, their homological structures are the same, which means that they have of the same number and types singularities -black holes.

It is known that black holes arise as a result of gravitational collapse. This process, consists of different stages, but in each stage we have transition from one level of structural hierarchy of the matter to lower level of hierarchy.

.Appearing Black hole at the final stage of this process consists of the matter which has the lowest level of structural hierarchy or the Black hole consists the matter without structure. The matter without structure does not exist and in such case it should disappear. This implies that black hole filled with so-called "primary matter" which structural hierarchy level is minimal, in such a case, it must mean that the black hole is one version of vacuum or inside of black holes there is absolute emptiness(no vacuum) which means that inside the black hole the physical space does notexist. If this conclusion is correct, then how to explain the huge gravitational attraction of black holes? It is accepted that in time section the physical space is Riemann's manifold[2]. Density of mass distributed on this manifold, in each point, identified as curvature of this manifold in this point[2]. Betty numbers of this Riemann`s manifold are closely related to the curvature of this manifold [2].

The most curvature Riemann`s manifold has in points which are located near holes of this manifold. This implies that huge gravitational attraction have the area of the physical space replaced near horizon of the black hole and not the hole inside.

If we assume, that the black hole, obtained by gravitational collapse, is vacuum or hole topological point of view, than destruction of levels structural hierarchy matter at this process should end with full annihilation. This annihilation of matter must be obtained by the action operators of annihilation and creation. Likely this action has been activated during such a process . This view strengthens the idea of representation of operators annihilation and creation as a boundary operator of the chain complexes and as co-boundary operators of co-chain complexes.

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To the fracture theory of multilayered materials

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Abstract. In this report the estimation of the element's fracture algorithm of the multilayered material of the thermonuclear unit reactor shell type, is based on the force approach, which is in its turn based on the use of the stress intensity coefficient K_1.

Keywords: multilayered materials.
PACS: 62.20.F-

In the case of the crack exposure on the edge of the interface between diverse materials, a two-parametric criterion is used, formulation of which includes two material constants, $K_{IIC}^{(i)}$ and τ_s. The following is assumed: for each material made from multilayered composition the brittle fracture is characteristic; on the boarders of the joint of the diverse materials made from the multilayered composition, conditions of the joined deformations are fulfilled.
The algorithm of the brittle strength estimation of the multilayered constructions consists of two stages, each establishing the type of fracture (normal rupture of the layer or stratification), the type and the size of the fracture and the crack's further behavior (stop-growth). These stages are the following:

Stage 1. Surface crack of the normal rupture in the first layer. It is assumed that this surface crack of the normal rupture is entirely within the layer which contains it. The solution of the corresponding boundary value problem for n (n≥1) - layered material, was obtained and the quantity K1 was determined as the function of the length of the crack, external load, geometrical and physical-mechanical parameters. Here also conditions were found at which the arrest (braking) of the crack occurs, other values of the parameters being equal. These conditions should be taken into account when designing and operating the multilayered constructions.
Stage 2. Surface crack of the normal rupture with its apex on the section's border. Two possible fracture cases are considered here:
Material stratification. Material stratification can take place if the condition $K_{IIC}^{(1,2)}$ < $K_{IC}^{(2)}$ is satisfied;

Propagation of the normal rupture crack in the second layer. This case is true when $K_{IC}^{(2)} < K_{IIC}^{(1,2)}$.

Operations presented above can be carried out multiple times, which allow describing the process of the crack propagation from one material into another.

For practical purposes while working out the analysis algorithm of the elements, of the multilayered constructions on the fracture, it was taken into account that:

The maximum use of the positions of the document ISDC (ITER Structure Design Criteria). Possibility of an experimental estimation of the defect size crack and its location within the multilayered material. Availability of the data on physical-mechanical material properties, on parameters of the brittle fracture of the multilayered composition $K_{IC}^{(i)}$ and on the adhesion strength on the boarders of the section $K_{IIC}^{(i,j)}$ or the possibility of their experimental determination.

Fracture stoppage, perpendicular to the interface of the two elastic environments

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Abstract. The main perpendicular fracture stoppage mechanism in the composite materials is the formation of the sliding fractures, which occur on the interface of the different elastic environments with its intersection by the main fracture of the normal separation.

Keywords: multilayered materials.
PACS: 62.20.F-

This mechanism is analysed below on the basis of the exact solution of the generalized Zak-Williams problem, found by the modified Wiener-Hopf method.

It is assumed that the length of the sliding fracture is small in comparison to the length of the main separation fracture and the typical body size. In this case the Zak-Williams solution presents itself as the precise asymptotics of the solution obtained on the distances, which are larger than the sliding fracture's length, but smaller than the length of the main separation fracture. Precise closed formulas are obtained for the tensions on the edge of the fracture and for the tension intensity coefficient on the edge of the sliding fracture as well.

1. *Problem definition.* Let's consider two homogeneous isotropic half-spaces from the different elastic materials, firmly coupled along the plane x=0 (diagram 1). Fracture's front coincides with the coordinate's origin. Problem is considered to be flat.

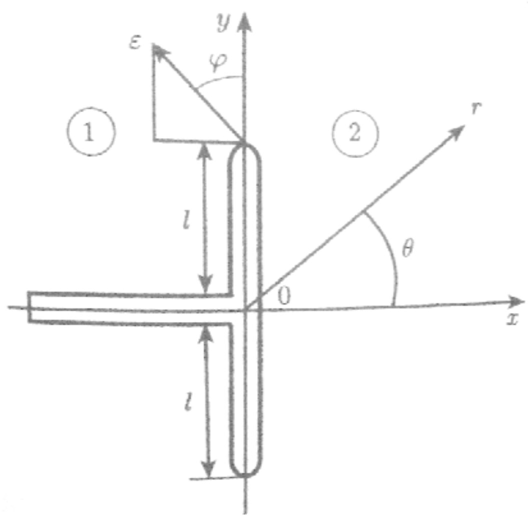

Diagram 1

Half-space *x≤0* is divided in two parts by the main separation fracture *y=0, x≤0*, banks of which are free from tensions. On the edge of half-spaces division there is a diametrical sliding fracture, length *l*, which is without generality restrictions, can be taken to be equal to one *(x=0, |y| ≤ 1)*. In case if the length *l* of the sliding fracture is equal to zero, canonical singularity problem results, which was precisely solved by Zak-Williams. In the task considered the Zak-Williams solution must realize as a set asymptotics of the desired solution with *r*→∞ (*r*, *θ* – polar coordinates).

Solution to the given problem is created. The length of the sliding line is determined

$$l = \left[\frac{\mathrm{E}sF_o(k)}{K_I F_1(k)} \right]^{1/\lambda}$$

Here $F_j(k)$ $(j=0.1)$ – known function (look []); K_I – tension intensity coefficient, which is s set load parameter in the considered singularity problem; λ – root of the Zak-Williams characteristic equation; quantity τ_s is constant (it characterizes resistance to the dislocation of the adhesion layer in the limit state). $\frac{1}{2}F_o(k), F_1(k), -\frac{1}{2}\lambda(k)$

Graphs of the functions, are shown on the diagram.

To the theory of the multilayered materials destruction with fracture

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Abstract. The estimation of the element's fracture algorithm of the multilayered material of the thermonuclear unit reactor shell type, is based on the force approach.

Keywords: multilayered materials.
PACS: 62.20.F-

Let's consider a problem. Let the elastic semiplane $x \geq 0$, made of $n+1$ various materials G_k, ν_k (G_k - the shift module, ν_k - factor of Puasson), rigidly linked along a plane x $x=h_k$, $k=1,\ldots,n$, contains a regional crack $y=0$, $0 \leq x \leq l$, $l<h_1$, which is perpendicular the border which is free of pressure $x=0$ of sections $x=h_k$, $k=1,\ldots,n$. Set normal tension is applied to the borders of the fracture . The problem is considered to be symmetric when concerns a plane $y=0$. On infinity pressures are absent, and displacements disappear.
Boundary conditions look like this:

$$x=0, \ \ |y|<\infty, \ \ (\sigma_x)_1=(\tau_{xy})_1=0; \tag{1.1}$$

$$y=0, \ \ x\in(0,l), \ \ (\sigma_y)_1=-p(x)\leq 0, \ \ (\tau_{xy})_1=0; \tag{1.2}$$

$$y=0, \ \ x\in(l,h_1), \ \ (\tau_{xy})_1=0, \ \ (v)_1=0; \tag{1.3}$$

$$y=0, \ \ x\in(h_{j-1},h_j), \ \ (\tau_{xy})_j=0, \ \ (v)_j=0, \ \ j=2,\ldots,n; \tag{1.4}$$

$$y=0, \ \ x>h_n, \ \ (\tau_{xy})_{n+1}=0, \ \ (v)_{n+1}=0; \tag{1.5}$$

$$x=h_j, \ \ |y|<\infty, \ \ (\sigma_x)_j=(\sigma_x)_{j+1}, \ \ (\tau_{xy})_j=(\tau_{xy})_{j+1}; \tag{1.6}$$

$$(u)_j=(u)_{j+1}, \ \ (v)_j=(v)_{j+1}, \ \ j=1,\ldots,n.$$

At $y=0, \ x \to l+0$

$$\sqrt{2\pi(x-l)}(\sigma_y)_1 \sim K_I. \tag{1.7}$$

Here $p(x)$ – the set continuous function; K_I – pressures intensity factor, which is subject to definition.

For the problem solution in $x \geq 0, \ y \geq 0$ we search in the form [1, 2]: at $h_{j-1}<x<h_j$ ($j=1,\ldots,n$, $h_0=0$; j's environment : G_j, ν_j)

$$2G_j(u)_j(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\left[\left(k_jA_1^{(j)}-\lambda B_0^{(j)}-\lambda xB_1^{(j)}\right)ch\lambda x+\left(k_jB_1^{(j)}-\lambda A_0^{(j)}-\lambda xA_1^{(j)}\right)ch\lambda x\right]\cos\lambda yd\lambda+$$
$$+\sqrt{\frac{2}{\pi}}\int_0^\infty\eta\left(C_0^{(j)}+yC_2^{(j)}\right)e^{-\eta y}\sin\eta xd\eta,$$
$$2G_j(v)_j(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\lambda\left[\left(A_0^{(j)}+xA_1^{(j)}\right)ch\lambda x+\left(B_0^{(j)}+xB_1^{(j)}\right)sh\lambda x\right]\sin\lambda yd\lambda+$$
$$+\sqrt{\frac{2}{\pi}}\int_0^\infty\left[\eta C_0^{(j)}+(\eta y+k_j)C_2^{(j)}\right]e^{-\eta y}\cos\eta xd\eta; \tag{1.8}$$

at $x>h_n=H$ (*(n+1)*'s environment: G_{n+1}, ν_{n+1})

$$2G_{n+1}(u)_{n+1}(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\left[\lambda A_4+(k_{n+1}+\lambda x)C_4\right]\cdot e^{-\lambda x}\cos\lambda yd\lambda,$$
$$2G_{n+1}(v)_{n+1}(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\lambda(A_4+xC_4)e^{-\lambda x}\sin\lambda yd\lambda,\ \ k_j=3-4\nu_j. \tag{1.9}$$

The condition (1.5) is thus satisfied.
We look for the function $C_2^{(1)}(\eta)$ in the form of [1, 2]

$$C_2^{(1)}(\eta)=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)J_0(\eta t)dt, \tag{1.10}$$

where $\psi(t)\in C^1[0,l]$ - the new unknown function.

Substituting (1.10) into (1.8) and satisfying conditions (1.1), (1. 2_2), (1. 3_1) and (1. 4), we get:

$$C_m^{(j)}=0\ \ (m=0,2;\ \ j=2,...,n),\ \ \eta C_0^{(1)}+(1-2\nu_1)C_2^{(1)}=0,\qquad \lambda B_0^{(1)}-(1-2\nu_1)A_1^{(1)}=0; \tag{1.11}$$

$$2(1-\nu_1)B_1^{(1)}-\lambda A_0^{(1)}=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)\{I_0(\lambda t)-L_0(\lambda t)+\lambda t[I_1(\lambda t)-L_{-1}(\lambda t')]\}dt. \tag{1.12}$$

Here $I_k(x)$ and $L_k(x)$ - the modified functions of Bessel and Struve.

According to (1.8) – (1.11) and to a boundary condition (1.6) we come to the system of equations, the order of a matrix of which is equal to *4n* relative to *4n+1* unknowns: ψ(t), $A_m^{(j)}$, *j*=1,…,*n*, $B_m^{(j)}$, *j*=2,…,*n*, A_4, C_4, where *m*=0,1. And this system of the equations, appears from (1.12), (1.10) and (1.8), is algebraic relative to *4n* of the unknown functions, $A_m^{(j)}$, $B_m^{(j)}$, A_4 and C_4.. Hence, if functions, $A_m^{(j)}$, $B_m^{(j)}$, A_4 and C_4 present in the way of

$$A_m^{(j)}=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)a_m^{(j)}(\lambda,t)dt,\quad B_m^{(j)}=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)b_m^{(j)}(\lambda,t)dt,$$
$$A_4=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)a_4(\lambda,t)dt,\qquad C_4=\sqrt{\frac{\pi}{2}}\int_0^l t\psi(t)c_4(\lambda,t)dt, \tag{1.13}$$

then the system mentioned above is reduced to system *4n* the linear algebraic equations relative to *4n* of the unknown functions, $a_m^{(j)}$, $b_m^{(j)}$ (j=1,…,n; k=2,…,n; m=0, 1) a_4 and c_4 with the unknown parts on the right.

By means of the remained boundary conditions (1.2_1) and (1.3_2) we come to the integrated equation of the Fredholm's type of the 2nd sort.

$$\psi_0(x) = \frac{2}{\pi}\int_0^x \frac{p_0(\tau)}{\sqrt{x^2-\tau^2}}d\tau + \int_0^l \psi_0(\tau)K(x,\tau)d\tau, \qquad x\in[0,l],\ \ l<h_1,$$

$$K(x,\tau) = \int_0^\infty \lambda\tau\{\lambda a_0^{(1)}(\lambda,\tau)I_0(\lambda,x) + b_1^{(1)}(\lambda,\tau)[2v_1 I_0(\lambda x) + \lambda x I_1(\lambda x)] +$$

$$+\lambda b_0^{(1)}(\lambda,\tau)L_0(\lambda,x) + a_1^{(1)}(\lambda,\tau)[2v_1 L_0(\lambda x) + \lambda x L_{-1}(\lambda x)]\}d\lambda,$$

$$\psi(x) = \sigma\psi_0(x),\ \ p(x) = \sigma p_0(x),\ \ \sigma\equiv const \neq 0. \tag{1.14}$$

The pressures intensity factor coefficient is defined as this:

$$K_I = \sigma\sqrt{\pi l}\psi_0(\cdot) \qquad (l<h_1),$$

$$\psi_0(\cdot) = \psi_0(x, l/h_1, h_1/h_2, \ldots, k_{1,2}, k_{2,3}, \ldots, v_1, \ldots, v_n)|_{x=1},\ \ k_{j,i} = E_j/E_i.$$

1.2 °. Solution analysis. Thanks to the representation of required functions in the form of (1.13) a practical calculation possibility of the values $\psi_0(\cdot)$ for any number of layers n and combinations of materials properties and by that the solution of problems on optimal design of the multilayered constructions. The computer program is design for this in which the solution to the equation (1.14) is reduced to the solution of the system of the linear algebraic equations with an order *m*m*.

Let's consider the private cases of the common solution. Let n=3*;* $h_q=qh_1$ *(*q=2,3*);* $v_1=v_2=v_3=v_4$=0,3*;* $k_{4,3}$=1; 0,1; 10; 0 (if $k_{j,i}$=0, than v_j=0), $k_{3,2}$=1; 0,1; 10; $k_{2,1}$=1; 0,1; 10 and $p_0(x)=(l-ax)/l$, h_3=1. Results of numerical calculation of the dependence $K_I/(\sigma\sqrt{\pi h_1})$ from l/h_1 at the various values parameters of which are specified above are presented on fig. 1.

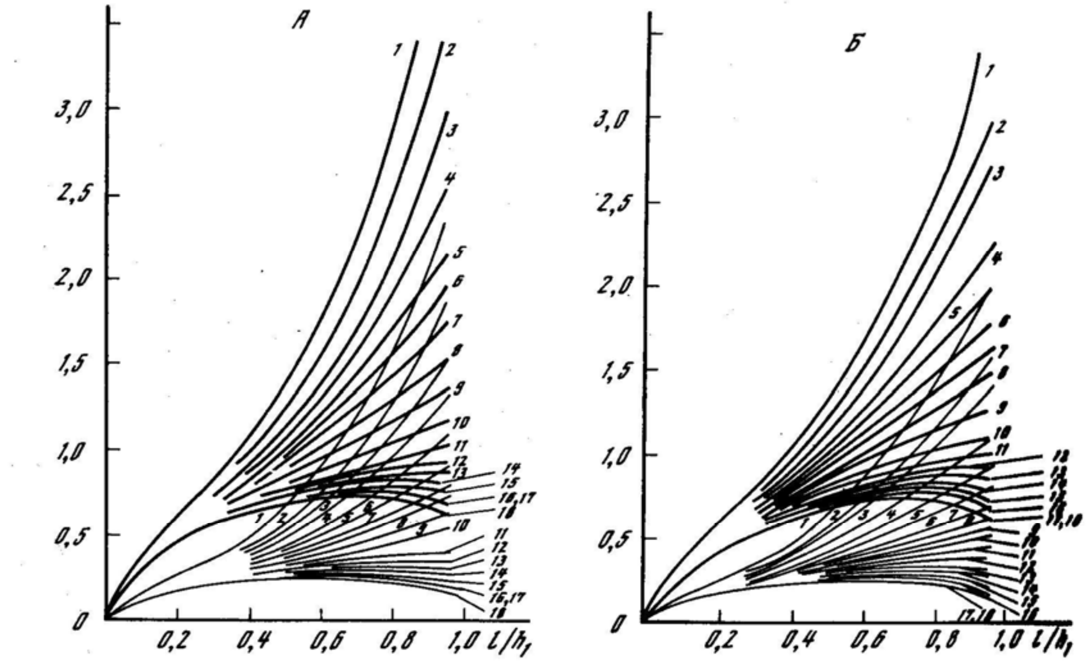

Рис. 1. Зависимость $K_I/(\sigma\sqrt{\pi h_1})$ от l/h_1. *А*: E_4/E_3 = 0 (*1–3, 7–9, 13–15*); 10,0 (*4–6, 10–12, 16–18*); E_3/E_2 = 0,1 (*1, 4, 7, 10, 13, 16*), 1,0 (*2, 5, 8, 11, 14, 17*), 10,0 (*3, 6, 9, 12, 15, 18*); E_2/E_1 = 0,1 (*1–6*), 1,0 (*7–12*), 10,0 (*13–18*); *Б*: E_4/E_3 = 0,1 (*1, 3, 5, 7, 9, 11, 13, 15, 17*), 1,0 (*2, 4, 6, 8, 10, 12, 14, 16, 18*); E_3/E_2 = 0,1 (*1, 2, 7, 8, 13, 14*), 1,0 (*3, 4, 9, 10, 15, 16*), 10,0 (*5, 6, 11, 12, 17, 18*); E_2/E_1 = 0,1 (*1–6*), 1,0 (*7–12*), 10,0 (*13–18*)

Thick lines represent the curves corresponding to a=0 (stretching), by thin lines – a=1 (bend).

During the calculations it was established that it is enough to have *m=20* in order to obtain the three steady decimal figures values of the function $\psi_0(\cdot)$.

The analysis shows: if $k_{2,1} < 1$, than at any values $k_{3,2}$ and $k_{4,3}$ function $\psi_0(\cdot)$ increases with the increase of $l/h_1 \in (0,1)$; if $k_{2,1} < 1$, than at the fixed values $l/h_1 \in (0,1)$ function $\psi_0(\cdot)$ increases with reduction of $k_{3,2}$; function $\psi_0(\cdot)$ at any fixed values of $l/h_1 \in (0,1)$ has the greatest value at $k_{2,1} = 1$, $k_{3,2} = 0$ and $k_{4,3} = 0$; if $k_{4,3} = 0$ and $k_{3,1} = 1$ than the function $\psi_0(\cdot)$ increases with increase of $k_{3,2}$ at any fixed values of $l/h_1 \in (0,1)$, thus if $k_{3,2} > 1$than the function $\psi_0(\cdot)$ increases with increase of $l/h_1 \in (0,1)$, and if $k_{3,2} < 1$ decreases with increase of $l/h_1 \in (0,1)$.

For the fracture indurance research of the multilayered materials, and also the processes of their destruction (fatigue failure, destruction at thermomechanical and radiating influences, etc.) the function $\psi_0(x, l/h_1, ...)$ approximations are very important, where $x \in (0,1]$. Function $\psi_0(x, l/h_1, ...)$ for the considered cases is approximated on a method of Chebyshev with a margin error in the parts of approximation no more than 0,01 % by the polynoms.

$$\psi_0(1, l/h_1, ...) = \sum_{q=0}^{4} d_q (l/h_1)^q, \qquad \psi_0(x, l/h_1, ...) = \sum_{q=0}^{5} p_q x^q, \qquad x \in (0,1].$$

Values of coefficients d_q and p_q because of the volume limitation are not specified here.

On the Generalized Short Pulse Equation Describing Propagation of Few-Cycle Pulses in Metamaterial Optical Fibers

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Abstract: We show that propagation of ultrashort (few-cycle) pulses in nonlinear Drude metamaterials with both electric and magnetic Kerr nonlinearities is described by coupled generalized Short Pulse Equations. The resulting system of equations generalizes to the case of metamaterials both the Short Pulse Equation and its vector generalizations which describe the few-cycle pulses in dielectric optical fibers beyond the slowly varying envelope approximation leading to the nonlinear Schroedinger equation.

Keywords: metamaterials, optical fibers, ultra-short pulses, few-cycle pulses, short pulse equation.
PACS: 78.67.Pt, 42.69.Tg, 42.65.Re, 05.45.Yv, 75.78.Jp.

Metamaterials are artificial structures that display properties beyond those available in naturally occurring materials. The most notable are the negative refraction index materials with simultaneously negative electric and magnetic dispersive responses, which affect substantially the conventional optics and its applications. Metamaterials with the negative refraction index have a number of extraordinary properties, such as the reversed Snell refraction, reversed Doppler effect, reversed radiation tension, negative Cerenkov radiation, reversed Goos-Haenchen shift, etc. Such materials can be used in various devices, such a compact cavities, superlenses, subwavelength waveguides and antennas, electromagnetic cloaking devices, tunable mirrors, isolators, phase compensators, and many others. Metamaterials can be also used to study ideas developed to describe physics in curved space-times and to model virtually any space-time metric of General Relativity.

A typical metamaterial with the negative refraction index is composed of a combination of a regular array of electrically small resonant particles, referred to as split-ring resonators, and a regular array of conducting wires responsible for the negative electric permittivity and negative magnetic permeability. The size and spacing of these elements is supposed to be much smaller than the wavelength of the propagating optical field, so that the metamaterial can be considered as a continuous and homogeneous medium.

Different models have been used to describe propagation of short and ultrashort optical pulses in metamaterials. For metamaterials without nonlinear magnetization a generalized nonlinear Schroedinger equation (NSE) has been derived [1, 2, 3, 4, 5]. For metamaterials with nonlinear magnetizations a single-component NSE for the electric field has been obtained in [6] . For the sufficiently long temporally optical pulses a system of coupled NSE can be used [7, 8, 9, 10].

However, for ultrashort few-cycle pulses the envelope approximation is not valid and the NSE cannot be applied. For metamaterials without the nonlinear magnetization a single-component generalized SPE for the electric field has been obtained in [11, 12]. When the nonlinear magnetization is present the equations for the electric and magnetic field cannot be uncoupled: Kinsler [13] derived in this case the equations for the unidirectional optical pulse propagation. Here we show that the propagation of few-cycle pulses in metamaterials with electric and magnetic Kerr-type nonlinear response can be described by a coupled system of Short Pulse Equations for the electric and magnetic field.

It has been shown that a composite metamaterial with negative refraction index can develop a non-linear macroscopic magnetic response. This means that, although the host medium has a negligible magnetic non-linearity, the periodic inclusions of the metamaterial can produce an effective magnetic non-linear response when the wave-length of the optical field is much larger than the periodicity of the inclusions, which is technologically feasible now.

Thus, let us consider a metamaterial with nonlinear magnetization. Let us start from the Maxwell equations for an optical field propagation along the *z*-direction in an optical fiber made of the corresponding metamaterial.

$$\partial_z E = -\partial_t B - \partial_t M_{nl}$$

$$-\partial_z H = \partial_t D + \partial_t P_{nl},$$

$$\partial_x D = 0$$

$$\partial_y B = 0$$

where it is assumed that the electric and magnetic fields are linearly polarized:

$$E = (E,0,0),\ H = (H,0,0).$$

The dielectric and magnetic response of a nonlinear material is characterized by the electric displacement field $\widetilde{D}(\omega) = \varepsilon(\omega)\widetilde{E}(\omega)$, magnetic induction $\widetilde{B}(\omega) = \mu(\omega)\widetilde{H}(\omega)$, nonlinear polarization $P_{nl} = \varepsilon_{nl} E$, and nonlinear magnetization $M_{nl} = \mu_{nl} H$.

Let us assume that both the electric and the magnetic nonlinearities are of Kerr type, $\varepsilon_{nl} = \chi_e E^2$ and $\mu_{nl} = \chi_m H^2$. Substituting the material equations into the Maxwell equations we obtain in the frequency domain:

$$\partial_z \widetilde{E} = -i\omega\mu(\omega)\widetilde{H} - i\omega\chi_m \widetilde{H}^3 \quad (1)$$

$$\partial_z \widetilde{H} = -i\omega\varepsilon(\omega)\widetilde{E} - i\omega\chi_e \widetilde{E}^3 \quad (2)$$

Acting with ∂_z on equation (1) and using equation (2) we get:

$$\partial_{zz}\widetilde{E} = -\omega^2\left(\mu(\omega)\varepsilon(\omega) + \mu(\omega)\chi_e\widetilde{E}^2 + 3\varepsilon(\omega)\chi_m\widetilde{H}^2\right)\widetilde{E}, \quad (3)$$

Similarly, acting with ∂_z on equation (2) and using equation (1) we obtain:

$$\partial_{zz}\widetilde{H} = -\omega^2\left(\varepsilon(\omega)\mu(\omega) + \varepsilon(\omega)\chi_m\widetilde{H}^2 + 3\mu(\omega)\chi_e\widetilde{E}^2\right)\widetilde{E}. \quad (4)$$

Now, let us assume that the dispersive properties of the metamaterial are given by the lossless Drude model:

$$\varepsilon(\omega) = \varepsilon_0\left(1 - \omega_e^2/\omega^2\right) \text{ and } \mu(\omega) = \mu_0\left(1 - \omega_m^2/\omega^2\right),$$

where ω_e and ω_m are the electric and magnetic plasma frequencies, respectively. Then

$$\varepsilon\mu = \varepsilon_0\mu_0\left(1 - \omega_e^2/\omega^2\omega_m^2/\omega^2 + \omega_e^2\omega_m/\omega^4\right).$$

Neglecting the term proportional to ω^{-4} and using $\varepsilon_0\mu_0 = c,$ where c is the velocity of light in vacuum, we obtain:

$$\varepsilon(\omega)\mu(\omega) = \varepsilon_0\mu_0\left(1 - \omega_e^2/\omega^2\omega_m^2/\omega^2\right).$$

Substituting these formulas into equations (3) and (4) and applying the Fourier transform, we obtain:

$$\partial_{zz}E = \partial_{tt}E/c^2 + \left(\omega_e^2 + \omega_m^2\right)/c^2E + \mu_0\chi_e\partial_{tt}\left(E^3\right) + \mu_0\omega_m^2\chi_eE^3 + 3\varepsilon_0\chi_m\partial_{tt}\left(H^2E\right) + 3\varepsilon_0\omega_e^2\chi_mH^2E, \quad (5)$$

$$\partial_{zz}H = \partial_{tt}H/c^2 + \left(\omega_e^2 + \omega_m^2\right)/c^2H + \varepsilon_0\chi_m\partial_{tt}\left(H^3\right) + \varepsilon_0\omega_e^2\chi_mH^3 + 3\mu_0\chi_e\partial_{tt}\left(E^2H\right) + 3\mu_0\omega_m^2\chi_eE^2H. \quad (6)$$

Introducing new variables: $\tau = t - z/c$ and $\zeta = z$ for which $\partial_{zz} = 1/2^2\partial_{\tau\tau} + 2/c\partial_{\zeta\tau} + \partial_{\zeta\zeta}$ and $\partial_{tt} = \partial_{\tau\tau}$. in equations (5) and (6) and making use of the paraxial approximation: $\partial_{\xi\xi}E = \partial_{\xi\xi}H = 0$, we obtain:

$$\partial_{\zeta\tau}E = \frac{\omega_e^2 + \omega_m^2}{2c}E + \frac{1}{2}\mu_0\chi_ec\partial_{\tau\tau}\left(E^3\right) + \frac{1}{2}\mu_0\omega_m^2\chi_ecE^3 + \frac{3}{2}\varepsilon_0\chi_mc\left(\partial_{\tau\tau} + \omega_e^2\right)H^2E, \quad (7)$$

$$\partial_{\zeta\tau}H = \frac{\omega_e^2 + \omega_m^2}{2c}H + \frac{1}{2}\varepsilon_0\chi_mc\partial_{\tau\tau}\left(H^3\right) + \frac{1}{2}\varepsilon_0\omega_e^2\chi_mcH^3 + \frac{3}{2}\mu_0\chi_ec\left(\partial_{\tau\tau} + \omega_m^2\right)E^2H. \quad (8)$$

Thus, we have obtained the set of two coupled generalized Short Pulse Equations. In the limit $\omega_m \to 0$ and $\chi_m \to 0$ equation (7) reduces to the Short Pulse Equation derived by Schäfer and Wayne [16]

$$\partial_{\zeta\tau} E = \frac{\omega_e^2}{2c} E + \frac{1}{2} \mu_0 \chi_e c \, \partial_{\tau\tau} (E^3),$$

which later was shown to be an integrable system [15,16]. With certain combination of the parameters of the metamaterial we can also obtain from equations (7) and (8) different integrable vector generalizations of the short pulse equation obtained by us [17] and other authors [18,19].

In conclusion, a consideration of the propagation of ultra-short few-cycle polarized optical pulses in the Drude metamaterial optical fibers with electric and magnetic Kerr nonlineratity leads to the coupled set of equations which generalizes the Short Pulse Equation. It allows to describe the ultra-short and spectrally broad optical pulses beyond the slow varying envelope approximation. It may open a possibility of studying a new class of optical phenomena in metamaterials when the spectral range of the optical field overlaps the regions with different signs of optical indices of the metamaterial.

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