Utrecht University

EMMEΦ

Determinism in General Physics and the Foundation of Quantum Mechanics

arxiv: 2010.02019 (5 oct. 2020)

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Moscow Institute of Physics and Technology

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Gerard 't Hooft

At large time and distance scales the laws of nature appear to be entirely deterministic.

But at the atomic scale, indeterminism seems to emerge:
quantum mechanics.

Whence this mysterious fact? Why are we unable to follow atoms and molecules more precisely when the evolve?

Copenhagen: *do not ask that question, just follow the rules and you get the best predictions that are possible.*

Alas, the predictions come in the form of probabilities.
Like the predictions of the weather.

Now, as in the case of the weather, we can search for microscopic laws that can explain the erratic behaviour, even if we will never do better than the statistical predictions.

We wish to explain where the statistical fluctuations come from.
Is there an underlying, deterministic set of laws? How can we find them?

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But they assumed a formalism for causality that one can question:
That's not causality as we use it in particle physics!

They assume statistical independence.

But I think something is happening that they did not foresee,
and it explains where the stochastic behaviour comes from!

Consider this clue:

an unstable particle, regardless whether it decays in nanoseconds of with lifetimes of billions of years, follows an *exponential decay law.* Can this be squared with determinism?

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Yes ! Just assume that our vacuum is filled with white noise. In practice, this white noise will be completely stochastic, yet we may well assume some deterministic random noise generating agent is responsible, such as: *vacuum fluctuations.*

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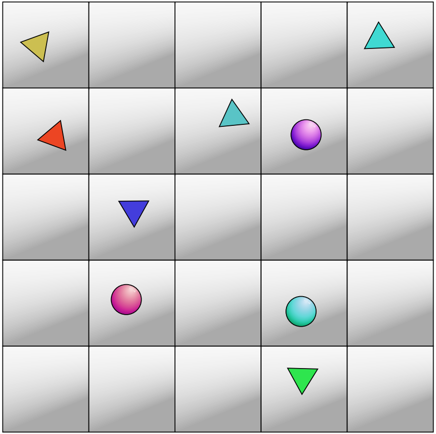
Apparently, we need a theory where the vacuum is actually a busy place:

A particle decays when its surrounding noise makes it decay, either rarely, or often. Can one construct models along such lines?

Yes! and much more. I'll show you how.

The Cellular Automaton: Only *classical* evolution equations.



(Quantum field lattice: same with *quantum* evolution equations)

Claim:

- Every cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.
- Every lattice quantum field theory can be accurately approximated by a *classical* cellular automaton.

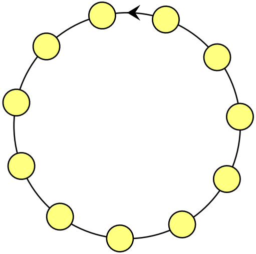
One needs to understand that every *classical* system can be described in the *quantum language* (Copenhagen) as if it were a *quantum system where the wave function does not spread in time*:

Hamiltonian linear in the momenta.

Operators are arranged in the following classes:

- Beables,
  refer to things that are 'truly there'.
  All beable operators commute with one another, at all times.
- Changeables,
  transform beables into other beables,
- Superimposables,
  all other operators.

But beware, this is only the first step.



1. The periodic chain.

Ontological states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

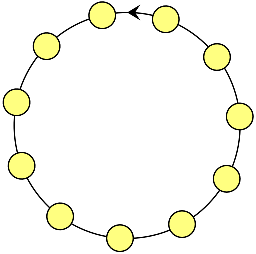
Evolution law:

$$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$$

$$U(\delta t)|k\rangle = |k+1 \mod N\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

Basic Ingredient for Models



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$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi i k n/N}|k\rangle^{\text{ont}}\ , \qquad k = 0,\cdots,\ N-1\ ;\quad n = 0,\cdots,\ N-1\ .$$

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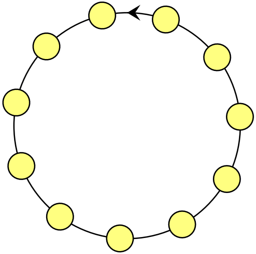

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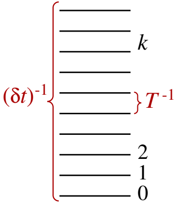
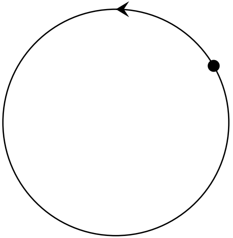
$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$





2. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$$

$$U(\delta t)|\phi\rangle = |\phi + \omega t\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

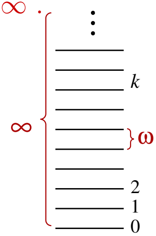
$$|n\rangle^E \ \stackrel{\mathrm{def}}{=}\ \frac{1}{\sqrt{2\pi}}\oint e^{i\phi n/N}|\phi\rangle^{\mathrm{ont}}\ , \qquad 0 \le \phi < 2\pi\ ;$$

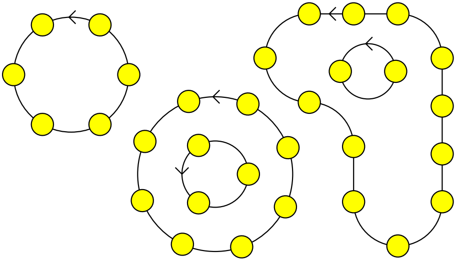
$$n = 0, \cdots, \ \infty\ .$$

$$|\phi\rangle^{\mathrm{ont}} = \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{-i\phi n/N}|n\rangle^E\ .$$

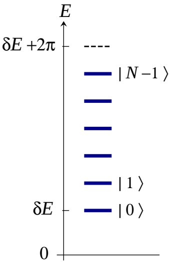
We generate exactly the spectrum of the harmonic oscillator:

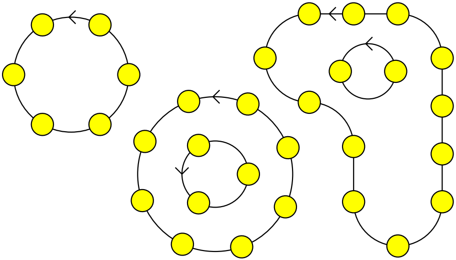
$$H = \omega\, n$$





Finite,
deterministic,
time reversible
models





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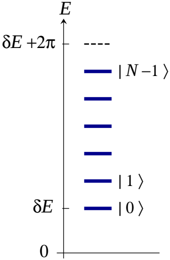

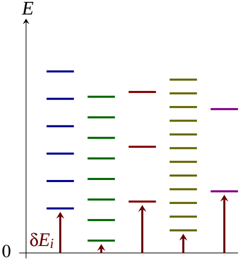

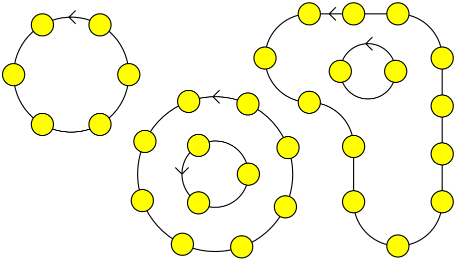

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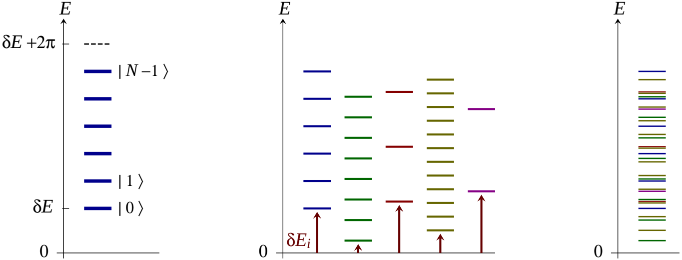

Since the time steps δt are discrete, ...

In the discrete case, the energy stays below a limiting value,
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There is a lowest energy state ("vacuum state"), and there is *a highest energy state* ("anti-vacuum")

possibly important in black hole physics,
where the horizon flips over the time coordidate

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But energy wasn’t a ‘beable’, it is a ‘changeable’. (see slide 7). Does this mean that energy is invisible in a deterministic theory?

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No !

In a theory with *fast fluctuating variables* (the white noise background), these variables, when followed variable by variable, would represent gigantic amounts of energy. But if we assume them to be ‘white noise’ then we can regard them, all taken together, to be in a zero energy (or very-low-energy) state. This energy cannot be *exactly* observed, but approximately. Thermodynamics will generate equipartition, that is, all states with very low energy per variable, will be strongly preferred.

But then, all other states of the system will become inaccurately defined. We don’t know what state the white noise is in, so we don’t know which state all other variables are in. We enter into the situation that is only too familiar in QM.

This *is* QM.

The situation that we need to describe occurs when there is a slight inaccuracy in the definition of *time*. Due to energy-time uncertainty,

$$\Delta t \cdot \Delta E \approx \hbar \ ,$$

we find that a slight smearing of time implies that we (have to) ignore the highest energy states.

We do this all the time in our experiments with high energy elementay particles :

We can only observe the low energy particles. To unravel the high energy, or very massive elementary particles, we need accelerators with energies that we cannot reach.

We cannot follow the fast variables!

Thus, curiously, in the classical limit – read: large scale limit – energy *does* become an observable. But it does not commute with our original beables.

This creates a new – and interesting – situation,
which can indeed occur in ordinary classical theories.

I describe what happens in more detail in:

See arxiv:2010.02019 .

There, I construct a completely classical model, which behaves *exactly* as a quantum system.

Here, I present an outline.

Consider a quantum system with a finite dimensional 'Hilbert' space of states. The Hamiltonian is an arbitrary, $N \times N$ hermitean matrix. We construct a model that will generate this matrix as an 'effective' or 'emergent' quantum Hamiltonian.

We assume N classical, fast variables, one for every state of the system.

> This sounds like a lot, but we are thinking of the vacuum fluctuations of a high-mass elementary particle. It has independent field degrees of freedom in every small volume element of space.
> This suffices, and we can economise later (multiple use of a given fast variable)

Each fast variable i lives on a circle with period L_i.

Take L_i discrete, like in our elementary unit model ($\rightarrow$ N-dimensional torus).

> Take the different L_i to be relative primes.

All periods L_i are shorter than the inverse mass of the fast fluctuating objects (particles).

e.o.m.: $x_i(t+1) = x_i(t) + 1 \mod L_i$.

This is driven by the Hamiltonian:

$$H = \sum_i p_i \ , \quad p_i = \frac{\partial}{\partial x_i} = \frac{2\pi n_i}{L_i} \ , \qquad n_i = 0, 1, \cdots, L_i - 1 \ .$$

Assume an *even distribution* of these variables. This means that, in our *formal* quantum language, they are all in their ground states. To make the distribution not even, we need the excited states, but their energies, are at least $2\pi/L_i$, which we take to be much larger than the energies of our quantum states.

The quantum degrees of freedom that I want to describe next, consist of N classical states.

We start with having no evolution at all there, so, for the classical states,

$$H_{\rm class} = 0.$$

Now consider two states, i and j. Assume that I want to add δH_{ij} to my Hamiltonian. There are three possible forms:

$$H = \alpha_1\sigma_1 + \alpha_2\sigma_2 + \alpha_3\sigma_3,$$

$$\text{with} \quad \sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

If we wanted $\alpha_1 = \frac{1}{2}\pi$, $\alpha_2 = \alpha_3 = 0$, then this can easily be seen to be a classical evolution:

Use $e^{\pm\frac{1}{2}i\pi} = \pm i$, to see that, if $H = \frac{1}{2}\pi\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, then

$$\varepsilon^{-iH} = -\begin{pmatrix}0 & i\\ i & 0\end{pmatrix} \;;$$

this is a classical flip-flop.
frequency 1 in the given time unit.

In our system of N fundamental states, we want much *lower* energies, *larger* time units, and different, non-commuting elements in the Hamiltonian: $|\alpha_i| \neq 0$, but $\ll 1$. That would give real QM.

To obtain such behaviour in our model, we now use the fast fluctuating variables x_i and x_j :
If both x_i and x_j take on special values, say $x_i = x_i^{(1)}$ and $x_j = x_j^{(1)}$, our system makes its classical flip-flop. Thus, we add to the Hamiltonian of the fast variables a term

$$\delta H_{ij} = \tfrac{1}{2}\pi\sigma_1^{ij}\,\delta(x_i - x_i^{(1)})\,\delta(x_j - x_j^{(1)}) \;. \tag{1}$$

Here, σ_1^{ij} is the flipping operator σ_1 acting on the two state system $|i\,\rangle,\ |j\,\rangle$ of the low energy system.

Our fast variables are in their lowest energy state, which takes the same value at all points $|x_i, x_j\rangle$.
We do as we always do in real qm: perturbation expansion.
Thus we use the expectation value of Hamiltonian (1) as the new effective Hamiltonian:

$$\delta H_{ij}^{\rm eff} \to \tfrac{1}{2}\pi\sigma_1^{ij} \cdot \tfrac{1}{L_i L_j} \ .$$

Indeed, classically, the flip-flop takes place after time $L_i\, L_j$.

One can also use σ_2^{ij} and σ_3^{ij}, the same way. Classically, these are slightly different flip-flop actions, as explained in the paper.

And now, we can repeat this for all other flipflops, to obtain a Hamiltonian

$$H^{\rm eff} = \sum_{i<j,\, a} \tfrac{1}{2}\pi\sigma_a^{ij} \cdot \tfrac{n_{ij,a}}{L_i\, L_j} \ .$$

For L_i and n_{ij} large enough, we can mimic this way almost any Hamiltonian for the slow system.

But we are dealing with a classical system. How can it be quantum mechanical at the same time?

If we keep the fast variables in their lowest energy mode, every conbination $\{x_i\}$ is equally possible. This generates apparently stochastic behaviour.
Energy conservation will forbid the very high energy excited states of the fast variables.

The energy eigen states of the slow variables are exactly as in any quantum theory, showing the relative abundance and the evolution process of the various possible states.

Hard question:

How can we accommodate for quantum interference effects?

What happens classically when we have destructive interference?
Imagine your favorite two slit experiment.
According to the math, we expect interference fringes when the two slits are open. But according to QM, the fringes disappear if we now close one slit:

Closing one slit should now allow particles to reach the dark spots. How can that be?

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Other way to ask this question: consider a computer model for interference. Particles are guided by the computer to go through one of the slits and then hit the screen.
Suppose that I now consider only those particles that went through slit #1. Do they hit the forbidden spots OR WHAT ??

Answer:

No. Those particles still produce the same interference pattern.

Is the model wrong?

No.

No. Those particles still produce the same interference pattern.

Is the model wrong?

No. Selecting in the computer all particles that went through slit #1 is different from putting a detector there. Instead, we made a selection in the distribution of the fast variables. Their distribution is no longer even – as if we were dealing with the interference of the zero energy mode and several of the higher energy excited states.

In the computer, we can do that. But in the physical world we cannot generate those excited energy states for the fast variable – it is as difficult as creating out of the vacuum fluctuations a new particle with $M \gg 1$ TeV.

The interference pattern that we now see describes the interference between the lowest energy states and excited states of the fast variables.

We didn't need the second slit to get this interference.

I think we hit upon the mechanism by which quantum behaviour emerges due to the action of fast fluctuating variables (such as vacuum fluctuations of the fields of heavy, virtual particles).

To summarise our result:

Even classical systems allow us to define energy, being the eigen value of the evolution operator, the Hamiltonian.

Energy is exactly conserved. But it does not commute with any of the (time dependent) observables (that I called 'beables').

If we use energy to project out the states we can observe, then operators describing those states also no longer commute.

This is the origin of non-commutativity in QM.

We only need to consider the largest energy modes of the fast fluctuating variables to realise this effect.

THE END

Thank you